ECON 210C

University of California, San Diego

Spring 2009

Problem Set II

(due on Mon. Apr 20$^{\text{th}}$)

Exercise I: Asset Prices and Accuracy of Log-Linearizations

Consider a pure endowment economy, where the representative agent makes his choices regarding consumption (C) and savings (B) solving the following problem:

$$\max_{\{C_t,B_t\}_{t=0}^{\infty}} = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$
$$\text{s.t. } Y_t + B_{t-1} \geq C_t + Q_t B_t,$$

where Q_t is the price of a risk-free discount bond delivering one unit of consumption in period $t+1$. The endowment Y_t is assumed to follow the exogenous process

$$\frac{Y_t}{Y_{t-1}} = \epsilon_t,$$

where $\log \epsilon_t \sim N(0, s^2)$.

1. Derive the first order conditions of the agent's problem and the market clearing condition, showing that in equilibrium the following condition holds:
$$Q_t = \beta E_t \left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma} \tag{1}$$
2. What is the value of Q in a perfect foresight steady state?
3. Solve equation (1) to find an exact expression for the bond price Q_t as a function of constants and/or variables that are observable at time t or earlier. [HINT: You may need to use the following relationship: if X follows a log-normal distribution, then $E(X) = Ee^x = e^{E(x)+\frac{1}{2}Var(x)}$].
   (a) What is the average value of Q? Is it the same as the steady state value computed in 2)? Why?
   (b) How does the variance of the endowment process (s^2) affect the bond price?
   (c) Find the equilibrium condition for the interest rate $i_t \equiv -\log Q_t$.
4. Suppose that neither you nor your computer could find the exact solution obtained in point 3 above. Log-linearize (1) around the steady-state, and obtain an approximate solution for the interest rate as a function of constants and/or variables which are observable at time t or earlier. How does the interest rate depend on the variance of the endowment process according to this approximate relation?
5. Assume that the standard deviation of the endowment process (s) could take values in the interval [0.01,0.1]. Comparing the expressions obtained in points 3c and 4, calculate the approximation error (in percentage points), first assuming that the agent is risk-neutral ($\sigma = 0$) and then assuming that the agent has a degree of risk-aversion $\sigma = 5$.

Exercise II: Monetary Rules in the New-Keynesian Model

Consider an economy whose output-gap ($\tilde{y}$), real money balances ($m-p$) and inflation (π) dynamics are described by the following equations

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^n\right) \quad (2)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\tilde{y}_t \quad (3)$$

$$m_t - p_t = \tilde{y}_t - \eta i_t + y_t^n, \quad (4)$$

where $\sigma > 1$, and r_t^n and y_t^n evolve exogenously and independently of monetary policy.
In addition, suppose the central bank adopt the following monetary policy rule:

$$m_t = \bar{m} + (p_t - \bar{p}) + u_t^m,$$

where $\bar{p} = \bar{m}$ is a price level target, u_t^m is an exogenous monetary policy shock following the AR(1) process $u_t^m = \rho_m u_{t-1}^m + \epsilon_t^m$ with $\rho_m \in [0,1)$ and ϵ_t^m white noise.

1. Describe in words where equations (2) - (4) come from.

2. Using the above equations, write a system of equations with (current and future) inflation and output gap as functions of exogenous variables.

3. What are the necessary and sufficient conditions for uniqueness of the solution?

4. Using the method of undetermined coefficients, find the response of output gap and inflation to a monetary policy shock.

5. Using (4), show how the interest rate responds to a monetary policy shock. Is there a liquidity effect? [HINT: Remember that y_t^n evolves independently of monetary policy].

Exercise III: New Keynesian Phillips Curve with Price Indexation (OPTIONAL)

Consider a world with staggered price setting and price indexation, so that the problem of a firm reoptimizing its price in period t can be written as follows:

$$\begin{aligned} \max_{P_t^*} \quad &= \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}[P_t^* \Pi^k Y_{t+k|t} - \Psi(Y_{t+k|t})]\} \\ s.t. \quad & \qquad Y_{t+k|t} = \left(\frac{P_t^* \Pi^k}{P_{t+k}}\right)^{-\epsilon} Y_{t+k|t}, \end{aligned}$$

where P_t^* is the price chosen by the firm at time t, which is updated at a rate Π until the next reoptimization occurs. $Q_{t,t+k}$ is the stochastic discount factor, $Y_{t+k|t}$ is the output of a firm that last reoptimized its price in period t and $\Psi(\cdot)$ is the cost function.

Moreover, the aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \tag{5}$$

1. Derive the first-order conditions of the firm's problem and derive an approximate (log-linearized) expression for p_t^* as a function of aggregate variables (i.e. prices and marginal cost).
2. Using (5), derive a log-linear expression for the evolution of inflation π_t as a function of $p_t^* - p_{t-1}$.
3. Using the results of the previous two points derive the New Keynesian Phillips Curve, relating aggregate inflation to aggregate marginal costs.