

Problem Set III
(due on Mon. Apr 27th)

Exercise I: Steady State Distortions and Subsidies

Consider an economy with monopolistic competition (and flexible prices) as the one analyzed in class. In that case, we have shown that in a competitive equilibrium the following condition holds:

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{MPN_t}{\mathcal{M}} \quad (1)$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ and $\epsilon > 1$.

Assume that the utility of the representative agent is given by

$$U(C, N) = \log(C_t) - N_t$$

and that the production function is given by

$$Y_t = N_t$$

where $0 < \alpha < 1$.

1. Compare (1) with the condition that would prevail in an economy with perfect competition.
2. Calculate the distortionary subsidy (τ) that is necessary to guarantee that the equilibrium with monopolistic competition is efficient.
3. Suppose the government is not subsidizing the economy. Compute the equilibrium consumption and hours worked as a function of their efficient counterparts.
4. Assuming $\epsilon = 6$ and using the utility function specified above, calculate the welfare loss due to monopolistic competition in terms of Consumption Equivalent Variation from the efficient level of consumption. How does the loss depend on the coefficient ϵ ? Explain.
5. Compare the welfare loss computed in the previous point with the loss associated with price stickiness in the presence of a suboptimal monetary policy rule (measures of the latter are reported in the tables shown in class).

Exercise II: Monetary Policy Rules, the Zero-Lower Bound and Interest Rate Inertia

Consider an economy where the following relations hold:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \quad (2)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \quad (3)$$

where u_t is an i.i.d. cost-push shock with $E(u_t) = 0$ and $Var(u_t) = \sigma_u^2$.

Suppose also that the representative agent's loss function is given by

$$\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{\kappa}{\epsilon} x_t^2 \right] \quad (4)$$

Part I: Optimal Policy under commitment

Suppose that the central bank operates under commitment and sets its policy to minimize the representative agent's loss function (4), subject to (3).

- Write the Lagrangean of the policymaker's problem, clearly indicating the initial conditions on the Lagrange multipliers. Derive the first-order conditions of the planner's problem and find the equilibrium evolution of output-gap (x) and inflation (π).
- Using (2) solve for the interest rate in $t = 0$ and determine what is the probability that the interest rate hits the zero lower bound (i.e. calculate $Prob(i_0 \leq 0)$).
- Is there interest rate inertia? If so, what is its source?

Part II: Inflation Targeting

Suppose that the central bank follows an inflation targeting rule $i_t = \rho + \phi_\pi \pi_t$.

- Solve for the equilibrium path of output-gap (x) and inflation (π) and find the value of ϕ_π that minimizes $var(\pi_t) + \frac{\kappa}{\epsilon} var(\tilde{x}_t)$ while guaranteeing the uniqueness of the solution.
- Calculate the probability that $i_t \leq 0$. Is this probability higher or lower than under the optimal policy with full-commitment computed above? (Assume that $\sigma\epsilon > 2$)
- Is there interest rate inertia in this case?
- (OPTIONAL) Repeat the exercise of the previous three points assuming that the central bank follows a price-level targeting rule $i_t = \rho + \phi_p(p_t - p^*)$ and assuming that $p_{-1} = p^*$.