

**Problem Set 4**  
 (due on Fri, Feb 10<sup>th</sup>)

**Exercise 1: An RBC model with preference shocks**

Consider an economy with a continuum of identical, infinitely lived agents, with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C^{1-\sigma}}{1-\sigma} - \frac{\exp(u_t)}{1+\theta} N_t^{1+\theta} \right] \quad (1)$$

where  $C$  is consumption,  $N$  is hours worked, and  $u$  is an exogenous taste parameter that follow an AR(1) process  $u_t = \rho u_{t+1} + e_t$ . Each consumer seeks to maximize eq. (1) subject to a sequence of budget constraint of the form

$$B_{t+1} + C_t = B_t(1 + r_t) + W_t N_t + \Pi_t, \quad (2)$$

where  $W_t$  is the wage,  $B_t$  denotes the holding s of a riskless bond, and  $\Pi_t$  are profits from ownership of the firms. The representative firm has access to a production function

$$Y_t = \exp(z_t) N_t^{1-\alpha}, \quad (3)$$

where  $z$  denotes an exogenous technology parameter following the AR(1) process  $z_t = \phi z_{t-1} + v_t$ . Answer the following questions:

1. Write down the equations that should be satisfied in a competitive equilibrium for this economy.
2. Determine the equilibrium process followed by  $y_t \equiv \log(Y_t)$ ,  $n_t = \log(N_t)$  and  $w_t = \log(W_t)$ .
3. Explain why the sign of the employment response to technology shocks depend on the size of  $\sigma$ .
4. Suppose that  $\sigma < 1$ . Can the model account for the correlation between employment and labor productivity observed in the data? Why?

**Exercise 2: Business Cycles and Government Expenditure**

Consider an economy where the representative agent makes his choices regarding consumption ( $C_t$ ) and hours worked ( $N_t$ ) solving the following problem,

$$\begin{aligned} \max_{\{C_t, N_t\}_{t=0}^{\infty}} &= E \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{N_t^{1+\phi}}{1+\phi} \right] \\ \text{s.t. } &W_t N_t \geq C_t - \mathcal{T}_t, \end{aligned}$$

where the sequences of wages  $W_t$  and lump-sum taxes  $\mathcal{T}_t$  are taken as given.

Firms produces output according to the following production function  $Y_t = N_t^{1-\alpha}$ . Finally, the government is providing a (useless) public good, financed through lump-sum taxes, as an exogenous proportion of current GDP. In particular  $G_t = \gamma_t Y_t$ , where  $\gamma_t$  evolves an exogenously according to the process

$$\frac{\gamma_t}{\bar{\gamma}} = \left( \frac{\gamma_{t-1}}{\bar{\gamma}} \right)^\rho \exp e_t, \quad (4)$$

where  $\bar{\gamma}$  constitutes the steady state value of the ratio  $G/Y$ ,  $0 \leq \rho < 1$  and  $e_t$  is an i.i.d. shock with zero mean. Finally, the resource constraint implies that in any period  $t$ ,  $Y_t = C_t + G_t$ .

Answer the following questions:

1. Provide a detailed definition of competitive equilibrium in this economy, clearly indicating all the equations that should be satisfied in that equilibrium. Is the competitive equilibrium Pareto optimal? Explain.
2. Solve for the steady state state of the above economy, and discuss the effect of an increase in the steady state  $G/Y$  ratio (i.e. an increase in  $\bar{\gamma}$ ) on  $Y$ ,  $C$  and  $N$ . Provide an economic explanations of your findings.
3. What are the effects of a (temporary) shock to  $\gamma_t$ ? According to this model, can government expenditure shocks account for the comovements between consumption, output and hours observed in the data?
4. Let's now consider an economy identical to the one described above, but where public expenditure is productive. In other words, now the production function is  $Y_t = \gamma_t^\alpha N_t^{1-\alpha}$ . What is the effect of  $\gamma_t$  on output, hours and consumption? Can the model account for the comovements observed in the data? Why (not)?