

ECON 210B  
The Indivisible Labor Model

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## Labor Markets in the Standard RBC model

### Main criticisms:

- ▶ Hours not enough volatile
- ▶ Wages too volatile and too correlated with output
- ▶ Wage elasticity of labor supply is too high
- ▶ Fluctuations Intensive margin vs Extensive margin  
*Model* *Data*

### Main approaches:

- ▶ Indivisible Labor model (Hansen (1985), Rogerson (1988))
- ▶ Search and matching frictions (Diamond (1982), Pissarides (1985))

# The Indivisible Labor Model

Hansen (1985), Rogerson (1988)

- ▶ Main Idea: Non-Convexities in Labor Supply
- ▶ Justifications:
  - ▶ Non - Convex Production Function (... a warm-up period)
  - ▶ Non - Convex Preferences(... need to drive to work)
- ▶ Extreme case: People either work full-time ( $h_t = h_0 < 1$ ) or don't work ( $h_t = 0$ ), i.e.  $h_t \in \{0; h_0\}$   
⇒ ... inelastic individual labor supply.
- ▶ The rest of the model is the same as in the benchmark case:
  - ▶ Preferences:  $u(c_t, h_t) = \log(c) + \theta \log(1 - h_t)$
  - ▶ Technology:  $F(k_t, h_t) = A_t k_t^\alpha h_t^{1-\alpha}$
  - ▶ Feasibility Constraint:  $c_t + k_{t+1} \leq F(k_t, h_t) + (1 - \delta)k_t$
  - ▶ Exogenous AR(1) process for  $A_t$ .

## The Indivisible Labor Model: Solution

**Problem:** Non - convexities in the problem  $\Rightarrow$  Individual choices not supported by a competitive equilibrium

**Solution:** Convexify the choice set. (... Many equivalent alternatives)

There is a continuum of individuals in the interval  $[0, 1]$ .... and a "lottery".

Timing of actions:

1. Individuals instead of choosing  $h_t$ , choose the probability  $p_t$  of working ("winning of the lottery").
2. Labor contract: Firm pays a hourly wage  $w_t$  to individuals who will work  $h_0$  hours with probability  $p_t$ .  
... the firm is providing unemployment insurance  $\Rightarrow c_t^w = c_t^u$
3. Uncertainty realizes, i.e. a lottery determines who works (mass  $p_t$ ) and who does not (mass  $1 - p_t$ )

In each period  $t$  the expected utility of an individual is

$$\begin{aligned}u(c_t, p_t) &= p_t[\log(c_t) + \theta \log(1 - h_0)] + (1 - p_t)[\log(c_t) + \theta \log(1)] \\ &= \log(c_t) + \theta \log(1 - h_0)p_t\end{aligned}$$

## The Indivisible Labor Model: Solution (cont'd)

The average (and aggregate) number of hours worked is  $h_t = p_t h_0$

... each individual receives  $w_t p_t h_0$  (regardless if she works or not)

Taking as given wages and interest rates, the individual problem is then

$$\begin{aligned} \max \quad & E_0 \sum_{t=0} \beta^t [\log(c_t) + \theta \log(1 - h_0) p_t] \\ \text{s.t.} \quad & c_t + k_{t+1} \leq w_t p_t h_0 + r_t k_t + (1 - \delta) k_t \\ & k_0 \text{ given} \end{aligned}$$

The “representative” agent problem is

$$\begin{aligned} \max \quad & E_0 \sum_{t=0} \beta^t [\log(c_t) - \underbrace{\theta \frac{\log(1-h_0)}{h_0}}_B h_t] \\ \text{s.t.} \quad & c_t + k_{t+1} \leq w_t h_t + r_t k_t + (1 - \delta) k_t \\ & k_0 \text{ given} \end{aligned}$$

The **aggregate** labor supply is given by  $w_t = B c_t$ .  $\Rightarrow$  infinite aggregate elasticity of Labor Supply.

# Main Results

from Hansen (JME, 1985)

Table 1

Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

Series	Quarterly U.S. time series <sup>a</sup> (55, 3-84, 1)		Economy with divisible labor <sup>b</sup>		Economy with indivisible labor <sup>b</sup>	
	(a)	(b)	(a)	(b)	(a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)	1.76 (0.21)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)	0.51 (0.08)	0.87 (0.04)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)	5.71 (0.70)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)	0.47 (0.10)	0.05 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)	1.35 (0.16)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)	0.50 (0.07)	0.87 (0.03)

<sup>a</sup> The U.S. time series used are real GNP, total consumption expenditures, and gross private domestic investment (all in 1972 dollars). The capital stock series includes nonresidential equipment and structures. The hours series includes total hours for persons at work in non-agricultural industries as derived from the *Current Population Survey*. Productivity is output divided by hours. All series are seasonally adjusted, logged and detrended.

<sup>b</sup> The standard deviations and correlations with output are sample means of statistics computed for each of 100 simulations. Each simulation consists of 115 periods, which is the same number of periods as the U.S. sample. The numbers in parentheses are sample standard deviations of these statistics. Before computing any statistics each simulated time series was logged and detrended using the same procedure used for the U.S. time series.