ECON 210B
Basic Overview of VARs

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Overview

What is a VAR?

- A vector autoregression, initially proposed by Chris Sims (1970s, 1980s)
- Possible uses of VARs
  - Forecasting
  - Estimating the responses to particular shocks
  - Estimating a "structural" model
- Advantage: ... it is a reduced form estimation
- Disadvantage: ... it is a reduced form estimation

Vector Autoregressions

$$Y_t = B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t,$$

where Y_t is a Nx1 vector of observed variables, and u_t is a Nx1 vector of unobserved shocks with

$$Eu_t = 0 \quad Eu_t u_t' = V \quad Eu_t u_{t-j}' = 0 \; \forall j \neq 0$$

- The estimates $\hat{B}$'s, $\hat{u}$'s and $\hat{V}$ can be obtained by OLS (or MLE, Bayesian estimation, etc.)

- Some estimation issues:
  - How many lags?
  - Are the series stationary? If not, what should we do?

The Moving Average (MA) representation

The VAR process can be re-written as

$$\underbrace{(I_N - B_1 L - ...B_p L^p)}_{\equiv B(L)} Y_t = u_t$$

If the $AR(p)$ process is invertible, it can be written as a $MA(\infty)$ process

$$Y_t = \underbrace{(C_0 + C_1 L + C_2 L^2 + ...)}_{\equiv C(L)} u_t$$

with $C(L) = (I_N - B_1 L - ...B_p L^p)^{-1}$.

An example ...

$\rightarrow$ The estimates $\hat{C}'s$ are easily obtained from the $\hat{B}$'s.

The effects of shocks to the economy

$$Y_t = C(L)u_t$$

The $C's$ indicate the response of the variables Y_t to the shocks u_t.

Problem: In general the different elements of u_t are correlated ...
$\rightarrow$ canNOT separate the effects of each shock

Solution: Let's define $\epsilon_t \equiv P^{-1}u_t$ such that $E(\epsilon_t \epsilon_t') = I_N$.

The matrix P satisfies

$$V = E(u_t u_t') = E(P\epsilon_t \epsilon_t' P') = PE(\epsilon_t \epsilon_t')P' = PP'.$$

Thus, our system becomes

$$Y_t = C(L)P \underbrace{\epsilon_t}_{P^{-1}u_t}.$$

The $C(L)P$ are called the impulse responses to the (orthogonalized) shocks ϵ_t.

Identification

Identification problem:

- There are $N(N+1)/2$ equations in

$$\hat{V} = PP'.$$

- ... but P has N^2 unknown elements.
- More unknowns than equations ... need some "restrictions" (i.e. extra equations)
- How many? At least $N(N-1)/2$!

Common types of restrictions

$$Y_t = C(L)P\epsilon_t$$

1. "Short-Run" Restrictions: impose restrictions directly on P.
   - Example 1a: P is lower triangular (Cholesky decomposition)
     Problem: ... the (arbitrary) order of variables in Y_t matters
   - Example 1b: Put the zeros in P according to some "theory"
2. "Long-Run" Restrictions [Blanchard-Quah (1989)]: restrictions on $C(L)P$
   - Example 2a: A Bivariate example
   - Example 2b: A more general case

Long-Run restrictions: A Bivariate example

Blanchard - Quah (AER, 1989)

$$Y_t = BY_{t-1} + u_t \qquad u_t = P\epsilon_t$$

$$Y_t \equiv \begin{pmatrix} \Delta y_t \\ x_t \end{pmatrix} \qquad \epsilon_t = \begin{pmatrix} \epsilon_t^d \\ \epsilon_t^s \end{pmatrix}$$

with $y_t \equiv$ output, $x_t \equiv$ unemployment.

Question: How many restrictions do we need to identify P?

Assumption: ϵ^d has no Long-Run impact on y_t,

$$\underbrace{\left(\sum_{j=0}^{\infty} C(j)\right) P}_{[I-B]^{-1} P} = \underbrace{\begin{bmatrix} 0 & d_{12} \\ d_{21} & d_{22} \end{bmatrix}}_{D}$$

$$[I-B]^{-1} P = D$$

Long-Run restrictions: A Bivariate example (cont'd)

Blanchard - Quah (AER, 1989)

How to find P?
We can estimate B and V. Then notice that

$$\underbrace{[I-B]^{-1}P}_{D}\underbrace{P'([I-B]^{-1})'}_{D'} = \underbrace{[I-B]^{-1}V([I-B]^{-1})'}_{S}$$

Using our restriction on D we have

$$\left[\begin{array}{cc} d_{12}^2 & \cdot \\ d_{12}d_{22} & d_{21}^2+d_{22}^2 \end{array}\right] = \left[\begin{array}{cc} s_{11} & \cdot \\ s_{12} & s_{22} \end{array}\right]$$

We can the obtain $d_11 = \sqrt{s_{11}}$ (impose a sign restriction !), $d_{22} = s_{12}/d_{12}$, $d_{21} = \sqrt{s_{22} - d_{22}^2}$ and

$$P = [I-B]D$$

Long-Run restrictions: A more general case

$$Y_t = B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t,$$

where $u_t = P\epsilon_t$.

- Say we want to find the effects of the last component of ϵ_t.
- Assumption: the last component of ϵ_t is the only one with long-run effects on the LEVEL of the first-variable of Y_t, i.e.

$$\underbrace{[I - B(1)]^{-1}}_{\left(\sum_{j=0}^{\infty} C(j)\right)} P = \underbrace{\begin{bmatrix} 0, ..., 0 & d_{12} \\ D_{21} & D_{22} \end{bmatrix}}_{D}$$

with $B(1) \equiv B_1 + B_2 + ...B_p$

- As before, we have

$$\begin{bmatrix} d_{12}^2 & d_{12}D_{22}' \\ d_{12}D_{22} & D_{21}D_{21}' + D_{22}D_{22}' \end{bmatrix} = \begin{bmatrix} S_{11} & s_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

After imposing a sign restriction, we can find d_{12} and D_{22} (last column of D), and then obtain $P_{.2}$ (last column of P) as

$$P_{.2} = [I - B(1)] \begin{bmatrix} d_{12} \\ D_{22} \end{bmatrix}$$

A Practical example: identification of Technology Shocks

Galì, AER, 1999

The baseline framework:

- 2 variables: x_t = log(output/hours), n_t = log(employment).
- Variable in first-difference in the VAR: $Y_t = [\Delta x_t, \Delta n_t]$
- Two types of shocks: non-technology and technology.
- Identification: technology shocks are the only ones with long-run effects on x_t

VOL. 89 NO. 1 GALÍ: TECHNOLOGY, EMPLOYMENT, AND THE BUSINESS CYCLE 261

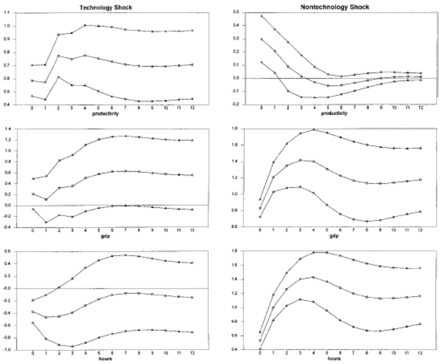

FIGURE 2. ESTIMATED IMPULSE RESPONSES FROM A BIVARIATE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)