Heterogeneous Pricing Behavior and Monetary Policy*

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PRELIMINARY

Abstract

This paper wants to study the determinants of the differences in pricing behaviour across sectors and analyze its consequences for the effects of monetary policy shocks. We build a two-sectors model allowing for several sources of heterogeneity among sectors, like price elasticity of demand, menu costs and sectoral shocks. The simulation of the calibrated model shows that the more competitive sector change its prices more frequently. Moreover, in our two-sectors model aggregate monetary policy shocks have more persistent aggregate effects than in the one-sector case. Persistency is even higher at a sectoral level and there are important differences in the response among sectors. We provide an example calibrating the model to match some features of the micro-evidence on pricing behavior in the Euro-Zone.

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1 Introduction

In the past decades, many structural models have been developed in order to analyze the effects of monetary policy on the real economy. Their common aspect is that the driving force of the non-neutrality of money is the inclusion of some source of rigidity, like stickiness of prices and wages. However, the results of such models crucially depend, among other things, on the specific way used to introduce these sources of rigidities, which reflects specific assumptions about the pricing behaviour of the agents in the economy. The plausibility of such assumptions should be evaluated on the basis of the empirical evidence. In this respect, in the last years a considerable effort has been spent to collect information on pricing behaviour, which provided a useful guide on how to introduce the pricing decision into models.

From the theoretical point of view, we can basically distinguish between two classes of models. On one side we have the so-called *time-dependent* models where, in the spirit of Calvo (1983), the timing of individual price changes is given exogenously, and the fraction of firms changing their prices is constant from period to period. In general, time-dependent models have the advantage of being simpler and more tractable, but they are able to account for the behavior of inflation only assuming implausible degree of pricing inertia, if compared to data at a micro-level [see e.g. Eichenbaum and Fisher (2004)][1]. In a standard calibration of the Calvo model with aggregate US data, the average duration of price spells is around 5 quarters while, as shown in the survey of Bils and Klenow (2004), in the period between 1995 and 1997, prices changed on average every roughly 5 months.

On the other hand, in state-dependent models [e.g Caplin and Spulber (1987)] firms are free to choose when to change their prices, but the repricing activity is costly. Thus, they will change prices only if the benefits

[1]See for example Eichenbaum and Fisher (2004), who show that Calvo (1983) model can account for post-war US inflation only introducing variable elasticity of demand and firm-specific capital.

of setting a new price outweigh the costs. In these models the frequency of price adjustment is endogenous and they therefore allow to match some features of the micro-evidence on pricing behavior. However, due to their technical complication, it is has been difficult to analyze their implications at the macro-level.

In this respect, the work of Dotsey, King and Wolman (1999) showed how the predictions of a *state dependent* dynamic general equilibrium model, in the case of a low-inflation economy were the frequency of price adjustment is roughly constant, closely resemble those of a more tractable *time-dependent* model. They therefore argue that this latter class of models are a good approximation of the more complicated, though more realistic, state-dependent models, thus suggesting that the inclusion of micro-founded pricing behavior into the problem could be of little advantage.

On the contrary, in a recent work Golosov and Lucas (2005), including firm specific idiosyncratic shocks, have built a model which is able to reconcile the evidence of low aggregate inflation and large size of individual price adjustment. They show, however, that in this framework money is nearly neutral, also in the short-run. This results is mainly due to the following reasons. First, firms when repricing in response to a idiosyncratic technology shock also adjust their price to incorporate the monetary shock. Second, they show how firms not repricing are those whose price is close to the optimum level, and thus the stickiness in those price have a small real effect. In other words they claim that what matters is not how many firms change their prices, as opposed to *time-dependent* models, but rather which ones. This result can thus be interpreted as undermining the microfoundations of New-Keynesian models, and stresses the importance of introducing a micro-founded pricing behaviour into models aimed to analyze the effects of monetary policy.

In this work we want to take into account another feature of the micro-evidence of pricing behaviour, and analyze its consequences for the effects of monetary policy. We observe that frequency of price adjustments differs systematically across sectors. As shown in table 1, for both the US and the Euro-zone countries[2], prices in the energy sector and in the unprocessed food sector change more frequently than those in the processed food or services sectors. Moreover, many empirical studies point out how these cross-sectoral differences are more relevant, in statistical terms, than cross-country differences.

There are many possible reasons beyond the presence of heterogenous pricing behavior: for example, differences in the price formation mechanism ("auction markets" vs. "customer markets"), differences in menu costs, sector specific shocks affecting either the supply or the demand side, or differences in the market power of firms belonging to different sectors. We try to include some of these possible sources of heterogeneity into our model. We then analyze their impact on pricing behaviour and, in turn, on the effects of a monetary policy shock in this framework.

We believe that being able to explain the differences in pricing behaviour across sectors can be useful for several reasons. First, to understand how the impact of monetary policy shocks differs across sectors, changing the relative prices of goods, and therefore creating distortions into the economy. This can be an additional source of persistence with respect to a standard one-sector model, and could be a step towards a reconciliation of the evidence on magnitude and persistence of monetary policy shocks with the micro evidence on price stickiness. Moreover, it is also possible that small and transient aggregate effects of monetary policy are the result of bigger and more persistent effects at a disaggregate level. Therefore, a model with many sectors can be important for the design of optimal monetary policy. As argued by Aoki (2001), in a two-sectors *time-dependent* model, it is optimal for the monetary policy authority to target sticky-price inflation, rather than a broad inflation

[2]The most comprehensive studies at a micro-level are those of Bils and Klenow (2004) for the US, and of Dhyne et al. (2005) for the Euro zone.

measure. However, studying optimal monetary policy in a framework with many sectors and endogenous stickiness is not the objective of this work, even if it could be a very interesting argument for future research.

We develop a model to endogenize the differences in pricing behaviour across sectors, therefore using a *state-dependent* model. We build on the framework recently developed by Golosov and Lucas (2005), in which monopolistic firms are affected by an aggregate shock (monetary policy), as well as by a firm-specific idiosyncratic shock. This implies that firms re-price at different point in time (as in the model of Dotsey, King and Wolman (1999), where firms were affected by different menu-cost shocks), and also that those firm changing price will generally set different prices. Our extension to the Golosov and Lucas model is the introduction of two sectors, with identical technologies but facing a different price elasticity of demand. Inside each sector there will be a continuum of firms, which are affected by idiosyncratic shocks to their marginal costs.

We first study how a different price elasticity of demand affects pricing behaviour, arguing that a different response of demand should affect the incentive of firms to change their prices, given that the repricing activity is costly. Using a discrete-time approximation to the continuous time problem [following Kushner and Dupuis (2001)], and a calibration of the relevant parameters on the US economy, we study how the average fraction of firms changing prices differs across sectors in a stationary equilibrium with constant inflation. We find that the sector with the higher price elasticity of demand is changing the price more frequently, and this difference is still present even for very high level of inflation. The economic intuition behind this fact is that in a dynamic context, for a firm with a low elasticity of demand (i.e. more market power), prices are relatively less important. On top of that it has more room to choose the price in order to reduce the probability of a costly future change.

This result is consistent with the empirical evidence found by Bils and

Klenow (2004), who show a negative effect of three different measures of market power on frequency of price adjustment. However, this result disappears when they control for whether goods are raw products, which are those affected by more volatile productivity. This suggests to extend our analysis to include sector specific supply-shocks. We incorporate this feature when we calibrate our model to match some of the stylized facts about the differences in pricing behaviuor for the Euro-Zone.

Moreover, we study the implication of heterogenous pricing behaviour for the transmission of monetary policy shocks. Our main findings are the following. First, monetary policy shocks have more persistent aggregate effects than in the one-sector case. Second, persistency is even higher at sectoral level. However, the overall effects are smaller and less persistent than in a standard time-dependent sticky prices model. Third, there are important differences in the response among sectors. Indeed, after an unexpected increase in the money supply, the price of the good produced by the sector with more market power becomes relatively cheaper. As a consequence, the share of such good in the consumption bundle increases. On the other hand, there are only small differences in the evolution of the profits among the two sectors.

Altogether, these results suggests that taking into account heterogeneities in pricing behaviour matters for the analysis of persistence of monetary policy shocks, and for their influences on sectoral production and employment. On the contrary, the difference in terms of profits is less pronounced, implying that the costs of inflation are similarly splitted among the sectors.

The paper is organized as follows. In Section 2, we present the model, its discrete-time approximation and the methodology we use to compute the responses to a monetary policy shock. In Section 3, we discuss our calibration procedure and expose our numerical results on the impact of market power on pricing behavior. We then turn to analyze the effects of a monetary policy shock on the economic variables. Finally, we calibrate the model to match some pieces of the micro-evidence on pricing behaviour in the Euro-Zone, and show the results for this case. Section 4 concludes.

2 The Model

As in Golosov and Lucas (2004)(GL from now on), the theoretical framework we use is a standard DSGE model with state-dependent pricing. Firms are affected by an aggregate shock, namely a monetary shock which is summarized in the money supply (M_t), and by a firm-specific idiosyncratic shocks to productivity (V_t). We assume the following processes for the log of money supply $m_t = \ln M_t$ and for the log of productivity $v_t = \ln V_t$:

$$dm = \mu dt + \sigma_m dZ$$

$$dv = -\eta v_t dt + \sigma_v dZ_v, \quad \eta > 0$$

where σ_m and σ_v are the variances, respectively, of money supply and technology shock, and Z_m and Z_v are two independent standard Brownian motion with zero drift and unit variance. In the rest of the paper I will only focus on the case of constant money growth at a rate μ (i.e. assuming $\sigma_m = 0$).

Moreover, firm sets their prices subject to a menu-cost of repricing: the presence of idiosyncratic shocks affecting productivity implies that not only do firms change their price in different periods, but that those that re-price will choose different prices.

We modify the GL model by introducing two different sectors, which differ only in terms of their price elasticity of demand.

2.1 Households

In this economy there is a continuum of infinitely-lived agents, whose utility is given by leisure, consumption and real money balances. Aggregate consumption is given by the combination of two types of goods, produced by two different sectors of the economy. Inside each of these there is a continuum of product varieties (or brands). We assume, for simplicity, that the two types of goods enter in the utility function in a Cobb-Douglas form, while we use the Spence-Dixit-Stiglitz specification to aggregate across the different varieties. In other words, aggregate consumption is

$$C_t = \frac{1}{\varphi^{\varphi}\left(1-\varphi\right)^{1-\varphi}} C_{1,t}^{\varphi} C_{2,t}^{1-\varphi} \tag{2.1.1}$$

The optimal allocation of any given expenditure between the two sectors will be:

$$C_{1,t} = \varphi \frac{P_t C_t}{P_{1,t}}; \quad C_{2,t} = (1-\varphi)\frac{P_t C_t}{P_{1,t}} \tag{2.1.2}$$

where the aggregate price $P_t \equiv P_{1,t}^{\varphi} P_{2,t}^{1-\varphi}$, and $P_{s,t}$, (for $s = 1, 2$) indicates the aggregate sector price, as detailed below. On the other hand, the consumption for the goods produced in each sector is given by:

$$\begin{aligned} C_{1,t} &= \left[\left(\frac{1}{\varphi}\right)^{\frac{1}{\theta_1}} \int_0^{\varphi} c_{s,t}(i)^{\frac{\theta_1 - 1}{\theta_1}} di\right]^{\frac{\theta_1}{\theta_1 - 1}} \\ C_{2,t} &= \left[\left(\frac{1}{1-\varphi}\right)^{\frac{1}{\theta_2}} \int_{\varphi}^{1} c_{s,t}(i)^{\frac{\theta_2 - 1}{\theta_2}} di\right]^{\frac{\theta_2}{\theta_2 - 1}} \end{aligned}$$

thus implying elasticities of substitution between goods in each sector of θ_1 and θ_2, respectively. We assume throughout that $\theta_1 < \theta_2$, that is sector 1 is the one with lower price elasticity of substitution. The optimal allocation across the varieties within the sectors will therefore be:

$$c_{1,t}(i) = \frac{1}{\varphi}\left(\frac{P_{1t}(i)}{P_{1,t}}\right)^{-\theta_1} C_{1t} \quad ; \quad c_{2,t}(i) = \frac{1}{1-\varphi}\left(\frac{P_{2t}(i)}{P_{2,t}}\right)^{-\theta_2} C_{2t} \tag{2.1.3}$$

where

$$P_{1,t} \equiv \left[\frac{1}{\varphi}\int_0^{\varphi} P_{s,t}(i)^{1-\theta_1}di\right]^{\frac{1}{1-\theta_1}} \quad P_{2,t} \equiv \left[\frac{1}{1-\varphi}\int_{\varphi}^{1} P_{s,t}(i)^{1-\theta_2}di\right]^{\frac{1}{1-\theta_1}}$$

We now turn to the intertemporal problem. Households' preferences over time are:

$$\left[\int_0^{\infty} e^{-\rho t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \alpha \ell_t + \ln M_t - \ln P_t\right) dt\right]$$

where ρ is the intertemporal discount factor, γ represent the risk aversion parameter, and the expectation operator is taken with respect of the two shock processes. Labor supply, ℓ_t, enters linearly in the utility function; this can be interpreted as indivisible labor with lotteries, following Hansen (1985). Finally the flow budget constraint is:

$$\overset{\circ}{M}_t + \overset{\circ}{B}_t = rB_t + W_t\ell_t + \Pi_t - P_tC_t$$

where B_t are bond holdings in period t, paying a constant nominal interest rate r[3], W_t is the nominal wage per hour worked, and Π_t is profit income, obtained from the households' holdings of a fully diversified portfolio of individual firms' shares. We have assumed that money balances pay a zero interest rate. Dots indicate the derivative of a variable with respect to time. After solving the Hamiltonian for the household's intertemporal problem, we obtain the following first order conditions:

$$e^{-\rho t}\alpha = \lambda_t W_t \tag{2.1.4}$$

$$e^{-\rho t}C_t^{-\gamma} = \lambda_t P_t \tag{2.1.5}$$

[3]The assumption of a constant interest rate implies that, in a stationary equilibrium with constant inflation, we have a constant level of aggregate consumption. This simplify our calculation, as explained below.

$$r = -\frac{\overset{\circ}{\lambda}_t}{\lambda_t} \tag{2.1.6}$$

$$e^{-\rho t}\frac{1}{M_t} = -\overset{\circ}{\lambda}_t \tag{2.1.7}$$

These imply that with a constant inflation rate μ, the equilibrium (constant) interest rate will be[4]:

$$r = \rho + \mu \tag{2.1.8}$$

Moreover, we want to emphasize that given our assumption on the utility function (i.e. separability and linear disutility of labor), we have the following evolution for nominal wages[5]:

$$W_t = W_0 e^{\mu t}. \tag{2.1.9}$$

Combining these first order conditions with the optimal allocations indicated above, we obtain the demand for each firm within the two sectors as a function of its own price, aggregate consumption and sectoral consumption[6]. Indeed:

$$c_{1,t}(i) = \frac{1}{\varphi}\left(\frac{\alpha P_{1t}(i)}{\varphi W_t}\right)^{-\theta_1} C_t^{\theta_1(1-\gamma)} C_{1,t}^{1-\theta_1} \tag{2.1.10}$$

$$c_{2,t}(i) = \frac{1}{1-\varphi}\left(\frac{\alpha P_{1t}(i)}{(1-\varphi) W_t}\right)^{-\theta_2} C_t^{\theta_2(1-\gamma)} C_{2,t}^{1-\theta_2} \tag{2.1.11}$$

[4]Substitute for $\overset{\circ}{\lambda}_t$in (2.1.6) with its expression in (2.1.7). Then take logs and first difference with respect to time and you get the result in (2.1.8)

[5]Combining (2.1.4), (2.1.7) and (2.1.8)

[6]We obtain this expression using (2.1.3), substituting the expression for sectoral price $P_{s,t}$ given by (2.1.2), and finally substituting for aggregate price level obtained by combining (2.1.4) and (2.1.5). We write individual firm's consumption in this form so that firm's problem do not depend on aggregate and sectoral prices, which simplifies the simulation of the model.

2.2 Firms

A typical firm in a sector produces a differentiated good, with a linear technology, where the only input is labor[7]:

$$c_{s,t}(i) = V_t \ell_{s,t}(i) \qquad s = 1, 2 \tag{2.2.1}$$

Moreover, firms use k hours of labor for the repricing activity. At each date, a firm faces consumer demand $c_{j,t}(i)$, a nominal wage rate W_t and a firm specific productivity parameter V_t. Given the current price P, it can decide whether to change its price or not. If it leaves the price unchanged its profit level will be

$$c_{s,t}(i)(P - W_t/V_t),$$

while if it changes the price to $Q \neq P$, its profit will be

$$c_{s,t}(i)(Q - W_t/V_t) - kW_t.$$

Given current prices and shocks, a firm has to choose its pricing strategy, which means choosing a sequence of stopping times $\{T^i\}_{i=1}^{\infty}$and optimal prices $\{P^i\}_{i=1}^{\infty}$ in order to maximize the discounted sum of future profits. We can write the problem as a Bellman equation like:

$$\begin{aligned} \Phi_s(P, V, W) &= \max_T E[\int_0^T e^{-rt} c_{j,t}(P)\,[P - W_t/V_t]\,dt \\ &\quad + e^{-rT} \max_Q\,[\Phi(Q, V_T, W_T) - kW_T]]. \end{aligned}$$

It follows that, given an initial distribution for prices and shocks $\phi_{s0}(P, V)$, and the pricing strategy of the firms, we will have a sequences of distribution of prices and shocks $\{\phi_{st}(P, V)\}_{t=1}^{\infty}$ and of the number of firms changing prices $\{\Upsilon_{st}\}_{t=1}^{\infty}$ for the two sectors. Finally, in each period we will a have a

[7] We already incorporate the equilibrium condition that output equals consumption.

labor demand given by:

$$\ell_t = \sum_{s=1}^{2} \left[\int_0^1 \frac{c_{s,t}(i)}{V_{s,t}(i)} di + k\Upsilon_{st} \right]$$

2.3 Equilibrium

A formal definition of the equilibrium is given in the original GL paper. Here we just want to remind that, given the assumption of separable utility function and linear disutility of labor, we have that labor supply is infinitely elastic at a wage W_t. Therefore, demand and supply always meet. Moreover, we have already included in the previous sections the following market clearing conditions

$$M_t^s = M_t^d = M_t$$

$$B_t = 0.$$

From (2.1.8) and (2.1.9), we have that

$$e^{-rt} = e^{-\rho t} \frac{W_0}{W_t}.$$

Substituting the households' optimality condition into the firm problem, we obtain the following Bellman equations for the two sectors:

$$\begin{aligned} \Phi_1(P_{1,0}, V_0, W_0) \quad &= W_0 \max_T E[\int_0^T e^{-\rho t} \left(\frac{\alpha P_{1,0}(i)}{\varphi W_t} \right)^{-\theta_1} C_t^{\theta_1(1-\gamma)} C_{1,t}^{1-\theta_1} \left[\frac{P_{1,0}(i)}{W_t} - \frac{1}{V_t(i)} \right] dt \\ &+ e^{-\rho T} \max_Q [\Phi_1(Q, V_T(i), W_T) - k]]. \end{aligned}$$

and

$$\begin{aligned} \Phi_2(P_{2,0}, V_0, W_0) \quad &= W_0 \max_T E[\int_0^T e^{-\rho t} \left(\frac{\alpha P_{2,0}(i)}{(1-\varphi) W_t} \right)^{-\theta_2} C_t^{\theta_2(1-\gamma)} C_{2,t}^{1-\theta_2} \left[\frac{P_{2,0}(i)}{W_t} - \frac{1}{V_t(i)} \right] dt \\ &+ e^{-\rho T} \max_Q [\Phi_2(Q, V_T(i), W_T) - k]]. \end{aligned}$$

In section 3.1, we simulate the firm's problem in a stationary equilibrium with constant inflation. Our goal is to calculate how the frequency of price adjustment in the two sectors, depends on the values of the price elasticity of demand.

First, we note that we note that in a stationary equilibrium with constant inflation the value of aggregate consumption will be constant at a value C. Moreover, to facilitate comparisons, we assume that the parameter $\phi = \frac{1}{2}$, which imply that there is an equal share of expenditure in the two sectors.

Finally, we define the variable $x = \log P - \log W$, whose evolution is $dm = -\mu dt$, so that we can write the firm problem as:

$$F_s(x_0, v_0) = \max_T E[\int_0^T e^{-\rho t}\Pi_s(x_0 - \mu t, v_t)dt + e^{-\rho T} \max_{x'} [F_s(x', v') - k]],$$

for s=1,2 and where

$$\Pi_s(x_t, v_t) = (2\alpha e^{x_t})^{-\theta_s} C_s^{1-\theta_s} C^{-\theta_s(1-\gamma)} \left[e^{x_t} - e^{-v_t}\right]$$

In the calculation reported below, we take a discrete approximation for time and states, i.e. a Markov chain, as described by Kushner and Dupuis (2001), choosing a discrete state space $S = X \times V$, and computing the transition probabilities

$$\pi(v', v) = \Pr\{v(t + \Delta t) = v' | v(t) = v\}$$

according to the parameters (η, σ_v). The discrete version of the continuous problem is

$$\begin{aligned} \psi_s(x, v) \quad = \quad & \max[\Pi_s(x, v)\Delta t + e^{-\rho\Delta t} \sum_{v'} \pi(v'|v)\psi_s(x - \mu\Delta t, v'); \\ & \max_{x'} \Pi_s(x', v)\Delta t + e^{-\rho\Delta t} \sum_{v'} \pi(v'|v)\psi_s(x' - \mu\Delta t, v') - k]. \end{aligned}$$

Using this approximation, we are able to solve the problem through dynamic programming, and then calculate the optimal pricing strategy for the firms, as a function of the current price and the shock. Moreover we are able to compute the time invariant distribution of the firms in the state space S, and the fraction of firms changing their prices (i.e. the re-pricing rate).

2.4 Impulse Response to a Monetary Shock

In this section, we describe the methodology we use to compute the responses of our variables to a one-time unexpected shock to the level of money M. In particular, we perturbate our economy in a stationary equilibrium with a constant level of inflation μ with an unanticipated jump in the level of money, after which money growth resumes its original rate μ. To do so, we compute the response of output in the two sectors, and then aggregate according to equation 2.1.1, to see the responses in the whole economy.

The technical complication to compute the impulse response functions is due to the presence of aggregate and sectoral consumption in the firm's problem, and the fact that we cannot solve it in a close form. Therefore, we follow a strategy similar to the one proposed by Golosov and Lucas, slightly modified to take into account the presence of two sectors.

We assume that the effect of the shock will be vanished after an arbitrarily large number of periods (say N), so that after N periods both aggregate and sectoral consumption are back at their steady state levels (C, C_1, C_2). We then guess the paths for consumption in the two sectors, $c_s^N = \{C_{s,t}\}_{t=1}^N$ $\forall s = 1, 2$, which by eq. 2.1.1 imply a path for aggregate consumption. In this way we are able to compute the sequence of value functions $\left\{\psi_{s,t}(x, v, c_1^N, c_2^N)\right\}_{t=1}^N$ by backward induction. Indeed we have:

$$\psi_{s,N}(x, v, c_1^N, c_2^N) = \psi_s(x, v)$$

and

$$\psi_{s,t}(x,v,c_1^N,c_2^N) = \max[\Pi_s(x,v,C_{s,t})\Delta t + e^{-\rho\Delta t}\sum_{v'}\pi(v'|v)\psi_s(x-\mu\Delta t,v',c_1^N,c_2^N); \\ \max_{x'}\Pi_s(x',v,C_{s,t})\Delta t + e^{-\rho\Delta t}dsum_{v'}\pi(v'|v)\psi_s(x'-\mu\Delta t,v',c_1^N,c_2^N)-k].$$

From these sequences of value functions we can recover the implied sequences of policy functions, so that we are able to find the pricing behaviour for the two sectors. Once we have the pricing behaviour, we can compute the sequences of joint distributions $\{\phi_{st}(P,V)\}_{t=1}^N$ for prices and technology shocks, taking as initial condition the joint distribution in the steady state. Thus, using 2.1.10 and 2.1.11 we obtain the sequences of consumption:

$$\Gamma C_{s,t} = \int \left(\frac{\alpha P_{st}(i)}{\varphi W_t}\right)^{-\theta_s} C_t^{\theta_s(1-\gamma)} C_{s,t}^{1-\theta_s}\phi_{s,t}(dP,dV) \qquad \forall s=1,2 \quad t=1,...,N$$

In other words, given our guess for the consumption paths, we are able to compute the implied equilibrium value for consumption in all the transition periods. We iterate this procedure until a fixed point is found, choosing a number of periods N such that the difference between $C_{s,N}-C_s$ is negligible.

3 Numerical exercises

We calibrate our model in two different ways. First, we choose the same calibration as in Golosov and Lucas, with the only difference that in our case the two sectors have a different degree of market power. This allows us to analyze how market power influences the frequency of price adjustment and to directly compare the predictions of our two-sectors model with the one-sector case.

In section 3.2, we turn to calibrate the relevant parameters to match some of the micro-evidence at sectoral level, as provided by the work of

Dhyne et al. (2005) for the Euro-area. In this way, we can analyze what are the implications of the model under a more realistic representation of the underlying economy.

3.1 The implications of different market power

In their work, Golosov and Lucas calibrate their model choosing standard values for the parameters indicating the intertemporal discount factor (ρ) and the coefficient of risk aversion (γ), a value for money growth rate (μ) equal to the average quarterly inflation during the period 1988-1997. Moreover they choose the values for the stochastic process of the idiosyncratic shock (η, σ_v) and for the number of hours required for the repricing activity (k), so that the simulated model is able to match some sample moments, like frequency of price adjustments, mean of price increases and standard deviation of new prices, using the data set of Klenow and Kryvstov (2003). The value of the parameter measuring the menu cost k implies that 0.5% of total employment is devoted to this activity, and menu costs represent about 0.5% of revenues[8].

Taking the same calibration, we conduct different experiments with the price elasticity of demand parameters, although restricted to be in a range between 2 and 20. Thus we only consider firms applying a mark-up between 5.2 and 100%[9]. The other parameters are summarized in table 2.

We conduct many simulations, where the low elasticity sector have a price elasticity $2 < \theta_1 < 20$, and for any value of θ_1 we let θ_2 to vary in a range between θ_1 and 20. In Figure **??**, we show the pricing behaviour of the two sectors in the cases $\theta_1 = 4$ and $\theta_2 = 10$ (upper panel) and $\theta_1 = 2$ and

[8]Levy et al. (1997) estimate that the cost of changing prices is about 0.7 percent of revenues.

[9]One can argue that a weakness of the model is related with the choice of the parameters θ_s. In order to have a "reasonable" firm's mark-up, we need $2 < \theta < 20$, which implies a mark-up in the range 5.2%-100%. Unfortunately, given the properties of the Dixit-Stiglitz monopolistic-competition model, this also implies that the demand drops by 2%-20%, for a 1% change in prices. The reasonability of this assumption at the firm level should be checked.

$\theta_2 = 20$ (lower panel), but the picture does not change for other values. The dotted line indicates the optimal price, while the two solid lines indicate the inaction region. In general, we can see that this region is narrower for high value of productivity, which implies that firms benefit more from repricing when productivity is high. Moreover, we can see that this fact is even more pronounced in the high elasticity sector.

When we calculate the time-invariant distribution over the set S, and the fraction of prices changed during one month, as shown in table 3, we find for the whole economy similar results to those obtained by Golosov and Lucas; in their simulation with only one sector and $\theta = 7$, they obtain a monthly re-pricing rate of 23.2% (i.e. an average price spell of 4.3 months). If we consider all our cases with aggregate elasticity of demand equal to 7, we have an aggregate frequency of price changes ranging between 22.7% and 25.2% (i.e. an average price spell of 3.9-4.4 months). All these data are consistent with the evidence on aggregate frequency of price adjustment for the US provided by Bils and Klenow (2004)[10]. In other words, our extension of the GL model, is able to account for some of the empirical evidence at the aggregate level.

However, when we look at the differences across the two sectors (see table 3), we observe that the sector with higher price elasticity of demand changes its price more frequently. The economic intuition behind this fact is twofold. First, with higher market power prices are less important, in the sense that prices have a smaller influence on demand, so that the gain of changing the price is smaller. Second, a firm with a higher market power, when changing its prices, can more freely decide its price to reduce the likelihood of a future change. In other words, with a higher market power a firm has more room to decide its price, and can do it to lower the cost of future changes.

Going back to our results, we can see in Table 3 that the frequency of

[10]Analyzing the period between 1995 and 2002, they find a median monthly frequency of price changes between 20% and 22%, and a median duration of spells between 4.0 and 4.6 months.

price changes varies between 0.8% and 31.0%. If we compare these results with the data in Table 1, we observe that the differences across our sectors are smaller than those in the data. In particular, with these range of values for θ, we are not able to account for such a low frequency of price adjustment in the services sector and such a high frequency in the energy sector. We think that including other source of asymmetries, like sector-specific supply shocks and different menu costs can help in this direction.

Figure **??** shows a simulated sample price path, for different values of firm market power affected by the same productivity shock, compared with the ideal price a firm would choose in absence of menu costs. We can see that firms with lower market power change their prices more frequently and that the size of price change is bigger the higher the market power. In addition, both the empirical studies of Bils and Klenow (2004) for the US, and Dhyne et al. for the Euro-zone, document a negative correlation between size of price changes and duration of price spell, which is what we can intuitively see in our simulated paths.

The relationship between frequency of price changes and market power has been recently investigated at the empirical level by Bils and Klenow (2004), who build a dataset matching micro data on prices of the BLS with data on market concentration from the Census of Manufacturing. They find that all the three measures of market power they use, namely market share of the 4 biggest firms within the sector and the average mark-up on marginal cost, affect negatively the frequency of price adjustment. However, this effect becomes insignificant once they control for whether a good is a raw product. Even if it seems reasonable that sectors selling raw products are characterized by a lower level of market power, this result suggests once more that we should conduct other experiments taking into account sector-specific supply shocks, given the peculiarity of shocks to raw products supply like vegetables or gasoline. A more complete calibration of the model, in order to match more closely the empirical evidence at sectoral level, is presented in section

3.2.

Finally, we have analyzed how the frequency of price changes is affected by changes in the stationary level of inflation. The results are shown in Figure **??**. First, we can see that when quarterly inflation goes from 0.64% to 10%, the frequency of price changes only increases by roughly 15% in the case of $(\theta_1, \theta_2) = (2, 12)$ and by only 8% with $(\theta_1, \theta_2) = (4, 10)$[11]. On the other hand, we can see that the differences across sectors are still present even with higher level of inflation. This strengthen the importance of including sectors with different degree of stickiness into models of monetary policy, for both high and low inflation countries.

However, the more interesting aspect is to analyze the response of our variables to a monetary shock. To do so we use the methodology described in section 2.4. We have tried to simulate the impulse response functions to a 2% increase in the level of money, for a two-sectors economy where the elasticities of substitution for the less and the more competitive sectors are $\theta_1 = 10$ and $\theta_2 = 4$ respectively, so that the average elasticity is 7, as in the GL paper.

First, let us focus on the aggregate effects, and compare our two-sectors economy with the GL's one-sector economy. In the upper panel of figure **??**, we compare our model with the extreme case of a one-sector model with only the stickier sector, i.e. with $\theta_2 = 4$. The initial response of real output is higher in the one-sector case. However, while in the one-sector economy the effect of the shock is over after about two quarters, in the two sector case the effect is more persistent, and it is still 0.05 percent higher than the initial level after three quarters.

If we turn our attention to sector specific effects (lower panel of figure **??**), we can see that persistence on sectoral output is even higher; after two

[11]The fact that frequency of price adjustment does not change so much for different level of inflation is one of the main critiques to $state - dependent$ models, given the modest gain in endogenizing price stickiness when analyzing economies with low and stable inflation (see for example Klenow and Kryvstov (2005)).

quarters, the high elasticity sector has an output about 0.1 percent higher than the initial level, while output in the other sector is about at the initial level. This fact can be explained looking at the path of relative prices[12] . Here we have a combination of two effects. On one side we have that the re-pricing rate is higher in sector 1, which should lead to a decrease of relative prices. On the other side, we have that firms in sector 1 have more market power, so that firms who are re-pricing can increase their price by more than the firms in the other sector, thus increasing relative prices. Thus, the evolution of relative prices across time depend on the strength of these two opposite factors. Immediately after the shock, goods in sector 1 becomes relatively cheeper, while later on they become relatively more expensive with respect to the initial level.

3.2 The effects of a monetary shock in the Euro-Area

We now turn to calibrate the parameters of our model in order to match more closely the empirical evidence on pricing behavior at a sectoral level. In this respect, we take as reference the empirical work of Dhyne et al. (2005), which reports data on pricing behaviour in the Euro-area, at the COICOP 5-digit level (see table 1). Given that the Services sector is the one with the lower frequency of price adjustment, and its share in the overall economy is quite high (36%), we decide to divide the economy into the services sector (Sector 2) and all the other sectors (Sector 1).

We set our parameters in the following way. We choose standard values for the parameters indicating the intertemporal discount factor (ρ) and the coefficient of risk aversion (γ). We then choose a value for the disutility of labor (α) such that the fraction of time devoted to labor is roughly 37% and a value for money growth rate (μ) equal to the average quarterly inflation during the period 1996-2001, namely 0.48%.

[12]We compute the relative price as the price in sector 1 over the price of sector 2, i.e. $\frac{P_{1t}}{P_{2t}}$, where the sector price is given by 2.1.3.

To choose the value for the parameter measuring the expenditure share in the two sectors we take the weight of the services sector in the CPI (namely 36%), so that we obtain a value of $\varphi = .64$. The parameters measuring the elasticity of substitution across product varieties within a sector (θ_1, θ_2) are chosen to match the estimates of average markup at product level in many European countries shown in the work of Przybyla and Roma (2005), where the markup in the services sector is reported to be about 30%, while for the average of the other sectors is roughly 14%. Given that in our model the gross mark-up is given by the ratio $\frac{\theta_s}{\theta_s - 1}$ $s = 1, 2$, these estimates imply values for θ_1 and θ_2 of about 4 and 8, respectively.

Finally, we need to choose values for the parameters for which we do not have empirical evidence, i.e. the parameters of the stochastic processes of the idiosyncratic shocks (η_s and $\sigma_{v,s}$,$s = 1, 2$) and the number of hours required for the repricing activity (k)[13]. We calibrate these parameters so that the simulated model, in stationary equilibrium, is able to match some sample moments like the frequency of price adjustments and mean of price increase of the two sectors. The values chosen and the corresponding moments are summarized in table 4.

The value of the parameters measuring the menu cost k implies that 0.5 of total employment is devoted to the repricing activity, which is roughly the same proportion as in the US economy.

We simulate the response of our variables to a one-period increase in the level of money, after which money keep growing at a level $\mu = .0048$ per quarter. Results are shown in figures **??** and **??**. When we compare the predictions of our two-sectors model with the one-sector case, we can see that although the response do not differ in terms of magnitude. On the other hand persistence is much higher in the two-sectors case. Indeed the effects of the monetary policy shocks are still sizeable after three quarters. Persistence is

[13]The parameter k measures the cost of repricing for a given firm. Given that we do not have evidence on differences in menu costs across sector, we have decided to choose a common value for the two sectors.

even higher for the variables at sectoral level. However, the overall effects are smaller and less persistent than in a standard time-dependent sticky prices model.

More interestingly, there are important differences among sectors for what concerns the responses of production and employment. Indeed, after an unexpected increase in the money supply, the price of the good produced by the sector with more market power (Services) becomes relatively cheaper. As a consequence, the share of such good in the consumption bundle increases, and consequently employment increases more than in the other sector. Overall, the costs of inflation, measured in terms of profits, are lower for the sector with more market power, even if the difference is less pronounced than for the other variables. This suggests that the costs of inflation are splitted similarly among the sectors. Finally, we should notice that, as in a standard two-sectors time-dependent model, the evolution of these variables contrasts with the empirical evidence found by Bils, Klenow and Kryvstov (2003). An investigation aimed at explaining this puzzle is an interesting line of future research.

4 Conclusions

We have extended the GL model to include two sectors with different price elasticity and using the same calibration of the original paper, we have simulated a stationary equilibrium with constant inflation. We have been able to replicate the basic result of the GL model, in terms of aggregate frequency of price adjustment, which are consistent with the empirical evidence found by Bils and Klenow (2004) for the US economy. Moreover, we found that firms in the high elasticity sector change their prices more frequently. Roughly speaking, this is due to the fact that prices are relatively less important when market power is higher, and demand is less elastic to prices.

When we compare our results with the empirical evidence at the micro-level, we can see that our model can account for most of the differences in frequency of price adjustment across sectors, with the exception of services and energy. Moreover, the relationship between market power and stickiness has been documented at an empirical level, although it is not robust to controls to sector specific components.

Moreover, we study the implication of heterogenous pricing behaviour for the transmission of monetary policy shocks. We calibrate the model to match some features of the micro-evidence on pricing behavior for the Euro-Zone, allowing for several sources of sectoral heterogeneity. Our main findings are the following. First, monetary policy shocks have more persistent aggregate effects than in the one-sector case. Second, persistency is even higher at a sectoral level. Third, there are important differences among sectors in the evolution of production and employment, while there are not important differences in the evolution of the profits. However, the overall effects are smaller and less persistent than in a standard time-dependent sticky price model.

These results suggests that taking into account the heterogeneity in pricing behaviour matters in these respects. First, for the analysis of persistence of monetary policy shocks. Second, to predict their influences on sectoral production and employment. On the other hand, the difference in terms of profits is less pronounced, implying that the costs of inflation are similarly splitted among the sectors.

References

[1] Aoki, Kosuke (2001). " Optimal Monetary Policy Responses to Relative-Price Changes.", *Journal of Monetary Economics*, 48:1 383-398.

[2] Bils, Mark, and Peter J. Klenow (2004). "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy*, 112: 947-985.

[3] Bils, Mark, Peter J. Klenow and Oleksiy Kryvstov (2003). "Sticky Prices and Monetary Policy Shocks" *Federal Reserve of Minneapolis Quarterly Review*, 27(1): 2-9

[4] Calvo, Guillermo A. (1983). "Staggered Prices in a Unitility Maximizing Framework." *Journal of Monetary Economics*, 12: 383-398.

[5] Caplin, Andrew S., and John Leahy (1991). "State Dependent Pricing and the Dynamics of Money and Output." *Quarterly Journal of Economics*, 106: 683-708.

[6] Caplin, Andrew S., and Daniel F. Spulber (1987). "Menu Costs and the Neutrality of Money." *Quarterly Journal of Economics*, 102: 703-726.

[7] Dhyne et al. (2004). "Price Setting in the Euro Area, Some Stylized Facts from Individual Price Consumer Price Data", mimeo

[8] Dotsey, Michael, Robert King, and Alexander L. Wolman (1999). "State Dependent Pricing and the General Equilibrium Dynamics of Money and Output." *Quarterly Journal of Economics*, 114: 655-690.

[9] Eichenbaum, Martin, and Jonas D.M. Fisher (2003). "Testing the Calvo model of sticky prices." *Economic Perspectives*, Federal Reserve Bank of Chicago, issue Q II), pages 40-53.

[10] Golosov, Mikhail, and Robert E. Lucas (2004). "Menu Costs and Phillips Curve" NBER Working Paper #10187.

[11] Klenow, Peter J. and Oleksiy Krystov (2003). "State-Dependent or Time Dependent Pricing: Does it Matter for Recent U.S. Inflation?" Federal Reserve Bank of Minneapolis working paper

[12] Kushner, Harold J, and Paul Dupuis (2001). *Numerical Methods for Stochastic Control Problems in Continuous Time.* Second Edition. New York: Springer-Verlag.

[13] Levy, Daniel, Mark Bergen, Shantanu Dutta, and Robert Venable (1997). "The Magnitude of Menu Costs: Direct Evidence from Large U.S. Supermarket Chains." *Quarterly Journal of Economics*, 113: 791-825.

[14] Przybyla, Marcin, and Moreno Roma (2005). "Does Product Market Competition Reduce Inflation? Evidence from EU Countries and Sectors." *ECB working paper series*, 453

[15] Taylor, John B. (1980) "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy*, 88: 1-24.

Table 1: Frequency of Price Changes by Product Type

	US	Euro
Unprocessed Food	47.7	28.3
Processed Food	27.1	13.7
Energy (oil products)	74.1	78.0
Non Energy Industrial Goods	22.4	9.2
Services	15.0	5.6
Total (weighted)	24.8	15.1

Source: Dhyne et al. (2004), p. 14

Table 2: Calibrated Parameters for the US

Param.	Value	Meaning	Criterion
γ	2	Degree of Risk-aversion	Standard
ρ	.01	Discount Factor	Standard
α	6	Disutility of labor	Implies ∼38% of time devoted to labor
μ	0.64%	Quarterly growth of money	Av. quart. inflation rate (88-97)
σ_v	.011	St. dev. of Technology shock	Match US micro-evidence
η	.55	Speed of mean reversion	Match US micro-evidence
k	0.0025	Menu cost	Match US micro-evidence

Table 3: Monthly Re-Pricing Rate for Sector 1 (in columns) (%)

θ^2 \ θ^1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	0.8	14.0	17.7	20.3	22.9	25.2	26.6	28.4	29.2	30.2	31.2	31.5	32.1	32.5	32.3	31.6	32.2	31.4	31.0
3	0.6	13.6	17.2	19.5	22.3	24.1	25.9	27.3	28.4	29.6	30.2	30.8	31.0	31.4	31.4	31.1	30.8	30.5	30.2
4	0.6	13.1	17.0	19.3	22.1	23.7	25.7	27.0	28.1	29.1	29.9	30.6	30.5	30.9	31.0	30.7	30.3	30.0	29.9
5	0.6	13.1	16.8	19.3	22.1	23.4	25.5	26.8	27.9	28.8	29.4	30.0	30.3	30.6	30.7	30.6	29.9	29.7	29.8
6	0.6	13.0	16.6	19.2	22.0	23.4	25.4	26.8	27.7	28.5	29.4	30.0	30.3	30.6	30.7	30.4	29.9	29.7	29.5
7	0.6	13.0	16.6	19.2	22.0	23.2	25.1	26.6	27.5	28.3	29.2	30.0	30.3	30.4	30.4	30.2	29.9	29.6	29.0
8	0.6	12.9	16.5	19.2	22.0	23.2	25.1	26.2	27.5	28.3	28.9	30.0	30.0	30.3	30.1	30.2	29.9	29.6	29.0
9	0.6	12.7	16.5	19.2	21.9	23.1	25.1	26.2	27.5	28.3	28.9	29.9	29.7	30.3	30.1	30.2	29.9	29.6	29.0
10	0.6	12.7	16.5	19.2	21.9	23.1	25.1	26.2	27.5	28.3	28.9	29.9	29.7	30.3	30.0	30.2	29.9	29.4	29.0
11	0.6	12.7	16.4	19.2	21.9	23.1	25.1	26.2	27.4	28.0	28.8	29.8	29.7	30.2	30.0	30.1	29.9	29.4	28.8
12	0.6	12.7	16.4	19.2	21.8	23.1	25.1	26.1	27.4	27.9	28.8	29.5	29.4	30.2	30.0	30.1	29.9	29.4	28.7
13	0.6	12.7	16.4	19.2	21.8	23.0	25.0	26.1	27.4	27.9	28.8	29.5	29.4	30.2	29.8	30.1	29.9	29.3	28.6
14	0.6	12.7	16.4	19.0	21.8	23.0	25.0	26.1	27.4	27.9	28.7	29.5	29.4	29.9	29.8	30.1	29.9	29.3	28.6
15	0.6	12.7	16.3	19.0	21.8	23.0	24.7	26.1	27.3	27.9	28.7	29.5	29.4	29.9	29.8	30.1	29.9	29.3	28.6
16	0.6	12.7	16.2	19.0	21.8	23.0	24.6	26.0	27.3	27.9	28.7	29.5	29.4	29.9	29.8	30.1	29.9	29.3	28.2
17	0.6	12.6	16.1	19.0	21.7	22.9	24.6	26.0	27.3	27.7	28.7	29.5	29.4	29.8	29.8	30.1	29.9	29.3	28.2
18	0.6	12.6	16.1	19.0	21.6	22.7	24.6	26.0	27.3	27.7	28.7	29.5	29.4	29.8	29.8	30.1	29.9	29.3	28.2
19	0.6	12.6	16.1	18.9	21.6	22.7	24.6	26.0	27.3	27.7	28.7	29.5	29.4	29.8	29.8	30.1	29.9	29.3	28.2
20	0.6	12.5	16.1	18.9	21.6	22.7	24.5	26.0	27.1	27.7	28.5	29.5	29.4	29.8	29.8	30.0	29.9	29.3	28.0

Table 4: Calibration of parameters for the Euro Zone

	Parameters			Data		Model	
Sector	k	η	σ_v	Price changes	Mean Increase	Price changes	Mean Increase
Non-Services	.0035	.88	.0103	19.2	9.1	19.4	9.0
Services	.0035	.5	.0025	5.6	7.3	5.3	7.3
Aggregate				15.1	8.2	14.4	8.4

Table 5: Parameters for the Euro-Zone

Param.	Value	Meaning	Criterion
γ	2	Degree of Risk-aversion	Standard
ρ	.01	Discount Factor	Standard
α	6	Disutility of labor	Implies ∼38% of time devoted to labor
μ	0.48%	Quarterly growth of money	Average quarterly Euro inflation rate (1996-2001)
φ	0.64%	Exp. share in the Non-Services	From Dhyne et al. (2004)
θ_1	8	Elasticity in the Non-Services	Markup ∼14% as in Przybyla and Roma (2005)
θ_2	4	Elasticity in the Services	Markup ∼30% as in Przybyla and Roma (2005)

Figure 1: Pricing Behaviour with 0.64 Quarterly Inflation

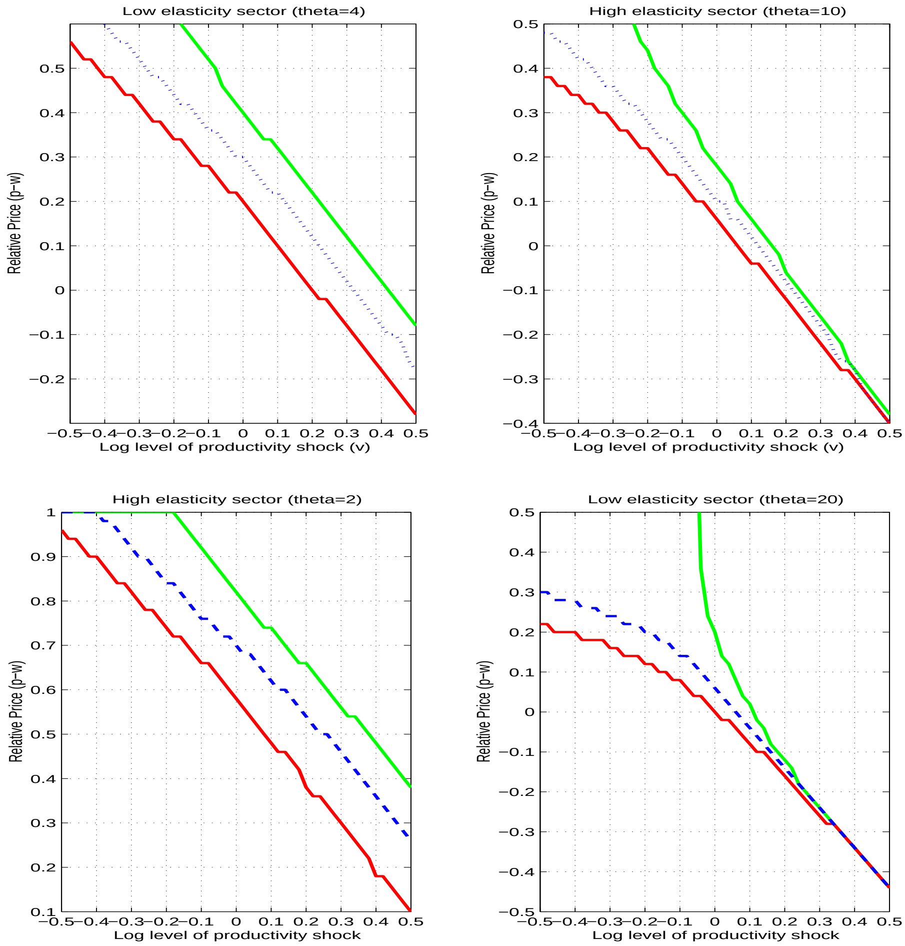


Figure 2: A Simulated Price Path

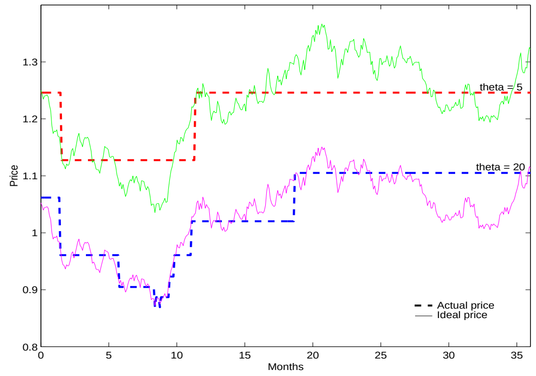


Figure 3: Frequency of price adjustment and inflation

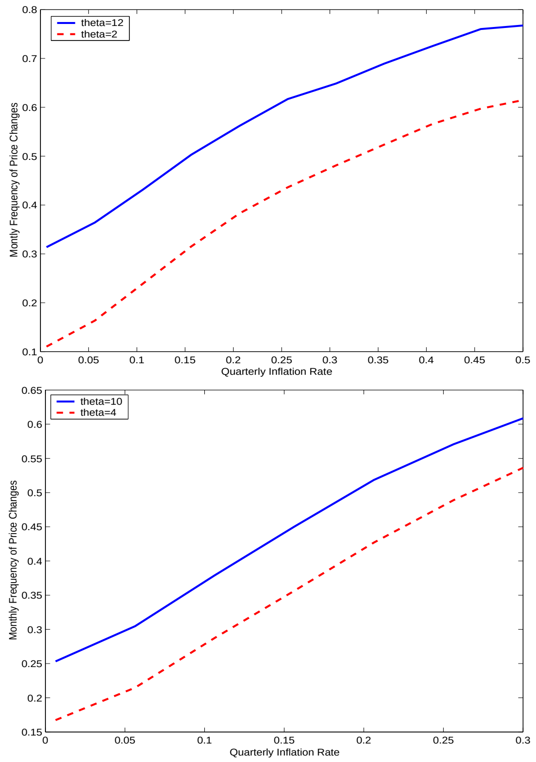
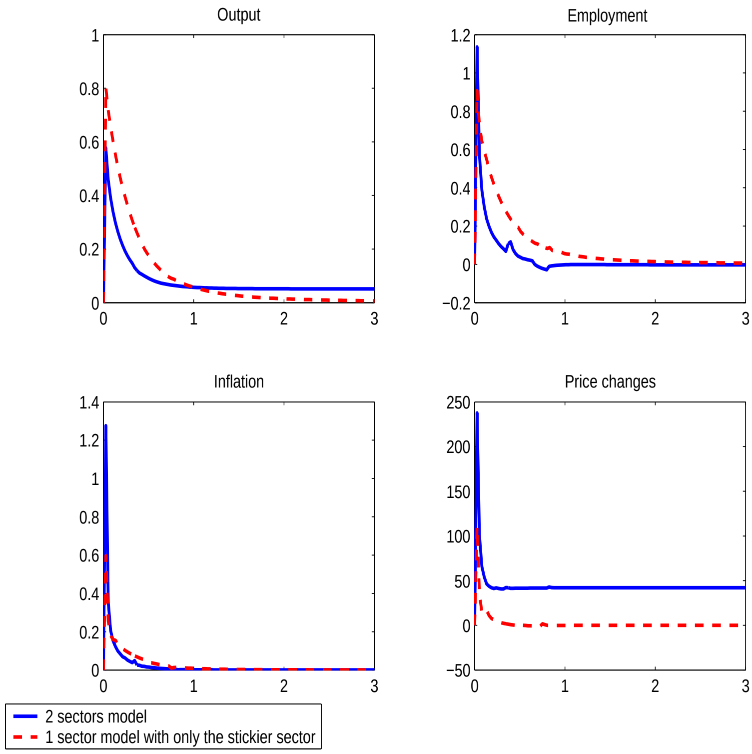


Figure 4: US - Impulse response to a 2% increase in the level of money.

A comparison with the one sector case



Low vs. high market power sector

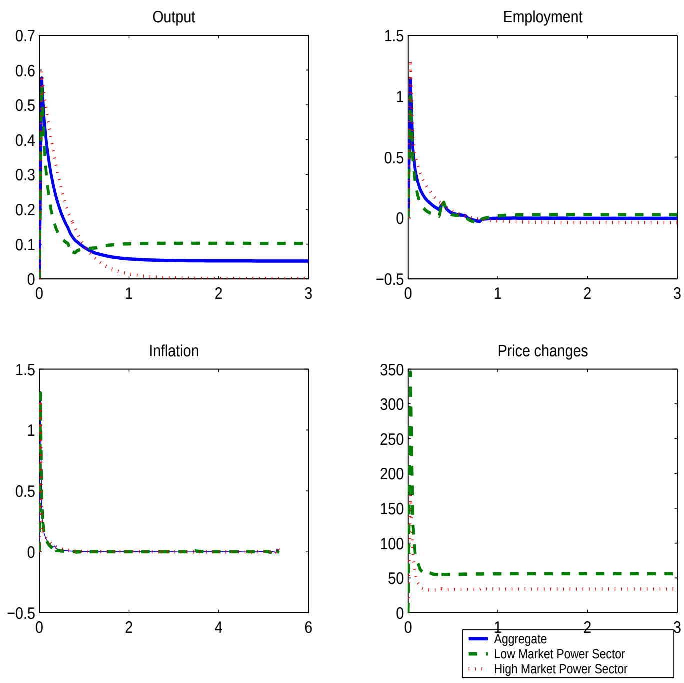
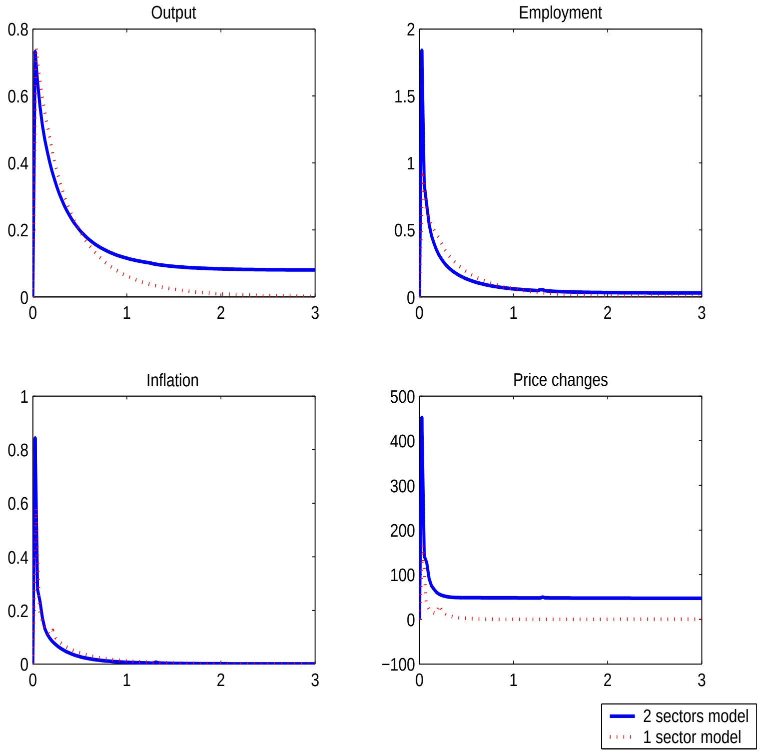


Figure 5: Euro-Zone - Impulse response to a 2% increase in the level of money.

A comparison with the one sector case.



Low vs. high market power sector

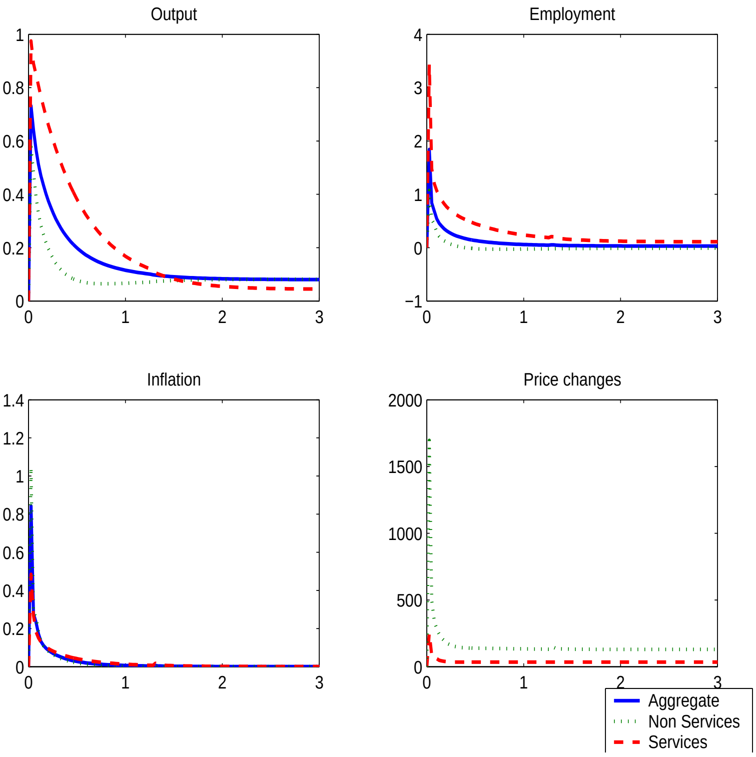


Figure 6: Euro-Zone - Impulse response to a 2% increase in the level of money.

Relative prices and output and period profits