Generation-1 Monetary Search Models

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Advanced Monetary Theory

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“On Money as a Medium of Exchange” (Kiyotaki and Wright, JPE 1989)

- Discrete time, infinite horizon.
- Unit mass continuum of agents.
- Three indivisible goods in the economy. Three types of agents (1/3 each): type i consumes good i, produces good $i+1$.
- All goods storable at a cost. Can only store 1 unit (hence 1 good) at a time.

Environment

- Storage costs c_{ij} of type i of agent: $c_{i3} > c_{i2} > c_{i1} > 0$.
- Lifetime utility:

$$\mathbb{E}\sum_t \beta^t [I_i^U(t)U_i - I_{i+1}^D(t)D_i - I_{ij}^C(t)c_{ij}]$$

- *Assumption:* $U_i - D_i > (c_{i,i+1} - c_{i,j})/(1-\beta)$, $\forall\ i,j$. *Net utility of consumption is high enough not to prefer autarky.*
- Whenever type i gets good i, he consumes it and instantly produces a new unit of $i+1$. $\rightarrow$ Each agent always has one good not of his own type.
- Trade: each period, agents are matched randomly in pairs. Decision: to trade or not to trade, bilaterally.
- Trade: one-for-one swap of inventories. No credit since the pair will meet again with probability 0.

Equilibrium Definition

- Let $p_{ij}(t)$ be the proportion of agents of type i who hold good j at date t. $\mathbf{p}(t)$ is the full distribution.
- Let $\tau_i(j,k) = 1$ if type i wants to trade j for k, 0 otherwise. Then, in an i-h meeting with goods j and k, trade occurs iff $\tau_i(j,k)\tau_h(k,j) = 1$.
- **Steady state Nash equilibrium**: a set of trading strategies $\{\tau_i\}$, $\forall i$, together with a steady-state distribution $\mathbf{p}$, such that: (a) each i chooses τ to maximize expected utility given all others' strategies and $\mathbf{p}$; and (b) given τ, $\mathbf{p}$ is the resulting distribution.

Optimal Trade

- V_{ij} - indirect utility (value) for type i of leaving a meeting with good j.

$$
\begin{aligned}
V_{ij} &= -c_{ij} + \max \beta \mathbb{E}[V_{ij'}|j] \\
V_{ii} &= U_i - D_i + \beta V_{i,i+1}
\end{aligned}
$$

- Optimal strategy: $\tau_i(j,k) = 1$ iff $V_{ik} > V_{ij}$.
- Note: $\tau_i(j,k) = 1$ iff $\tau_i(k,j) = 0$. Agents of same type never trade.
- *Lemma: Under the assumption above,* $\max_j V_{ij} = V_{ii}$.

Equilibria

The equilibrium will have the following properties:

1. *Fundamental equilibrium*: strategies described by $V_{ii} = \max_j V_{ij}$, $V_{12} > V_{13}$, $V_{21} > V_{23}$, $V_{31} > V_{32}$, $(p_{12}, p_{23}, p_{31}) = (1, 1/2, 1)$, if and only if $c_{13} - c_{12} > (\beta/6)(U_1 - D_1)$. Good 1 becomes medium of exchange, type 2 are middlemen.

2. *Speculative equilibrium:* strategies described by $V_{ii} = \max_j V_{ij}$, $V_{12} < V_{13}$, $V_{21} > V_{23}$, $V_{31} > V_{32}$, $(p_{12}, p_{23}, p_{31}) = (.5\sqrt{2}, \sqrt{2} - 1, 1)$, if and only if $c_{13} - c_{12} < (\beta/3)(\sqrt{2} - 1)(U_1 - D_1)$. Dual commodity monies, two types of middlemen. *Rate of return dominance*

3. In the intermediate region, no pure-strategy steady-state equilibria.

Fiat Money

- Introduce M units of good 0. No productive value (does not enter utility or production functions).

- Assume $c_{i0} = 0$. Suppose good 0 takes up a strictly positive amount of space such that one can *only* hold a unit of 0 or a unit of another good.

- A no-fiat equilibrium will exist when no one believes that money will be accepted, so that $V_{i0} = 0 < V_{ij}$, $j = 1, 2, 3$. *Tenuousness* of fiat currency.

Equilibrium with Fiat Money

- Suppose everyone has the belief that everyone else will accept good 0. Is this an equilibrium?
- Can show that $V_{ii} > V_{i0} > V_{ij}$, $j \neq i$, as 0 is both the lowest-cost and the highest-marketability good.
- Distribution $\mathbf{p}$ now depends on the amount of real balances in circulation.
- The equilibrium with fiat money exists (under some conditions) and looks like the fundamental equilibrium from before, with V_{i0} dominating other value functions. Commodity and fiat currencies co-exist, but fiat money is the only *general* medium of exchange (in that all agents accept it) - not so with the commodity money.

Liquidity

- A measure of liquidity of good j: the time it will take, on average, for agent i who holds j to trade it for his desired good.

- Let d_{ij} measure this duration.

- In equilibrium, for all i, liquidity depends on quantity of real balances S. As S rises, d_{ij} rises (too much money chasing too few goods).

- For all S, $d_{12} > d_{13} > d_{10}$, $d_{23} > d_{21} > d_{20}$, and $d_{32} = d_{31} = d_{30}$. III can always play fundamental, as can II, but I may sometimes speculate (in good 3).

"A Search-Theoretic Approach to Monetary Economics", Kiyotaki and Wright, AER(1993)

- What's different? - KW'89 is a model primarily designed to study commodity money. This is a framework better suited for fiat money.

- Address issues other than the rise of the medium of exchange: welfare properties of monetary vs. nonmonetary equilibria, interaction between specialization and monetary exchange, possibility of multiple currencies.

Environment

- Continuum of agents, measure 1. A continuum of consumption goods, real commodities, units of size 1.

- Fiat money - a non-consumable object.

- Specialization parameter: $x \in (0, 1)$, proportion of commodities that can be consumed by a given agent // proportion of agents that will consume a particular commodity.

- One's "own" consumption goods (within the consumption set) yield utility $U > 0$, else $U = 0$.

Environment

- At the start, $M \in [0, 1)$ agents endowed with money, $1 - M$ with real commodities, one per agent.
- Agents are endowed with 1 unit of real balances (so spend all the cash in any trade): assume money is indivisible. (Can also make it divisible and price the goods).
- Money and goods are costlessly storable. Money cannot be produced by agents.
- Production of real goods: inputs are consumption of own good and a random amount of time. Poisson process, arrival rate $\alpha > 0$ of production possibilities, once have consumed. Aka α measures productivity (average output per unit time).
- Cannot consume one's own output (relaxed later).

Exchange Sector

- Random pairwise meetings, Poisson process, arrival rate $\beta > 0$.
- No credit possible (0 probability of meeting the same person again). Only trade if mutually agreeable.
- Transaction cost $0 < \varepsilon < U$ for accepting a *real* commodity in trade.
- Assume transaction cost of accepting money is 0 (relaxed later, can let it have cost $0 \leq \eta < U - \varepsilon$, same effects).

Trade

- No agent can acquire more than one unit of good or money, since must give up one's entire inventory in any trade.
- An agent cannot produce until he acquires his own consumption good and consumes.
- Thus, if all start out with one unit of either a good or money, in equilibrium they will still have only one unit.
- Agents choose strategies on whether or not to accept money or various commodities, maximizing expected discounted utility from consumption net of transaction costs, taking as given strategies of others. Look for symmetric steady states.

Optimal Behavior

- An agent will always accept any commodity that is among the set of his own consumption goods.
- A *commodity trader* will never accept a commodity other than his own in trade, due to transaction costs and lack of additional benefits in terms of marketability in symmetric equilibria.
- Thus the parameter x measures the probability that a random commodity trader is willing to accept a given commodity (single-coincidence), and x^2 is the probability of double-coincidence.
- No commodity money in symmetric equilibrium.

Value Functions

- μ = the fraction of traders who are money traders ($1 - \mu$ are commodity traders).

- Π = probability that a random commodity trader will accept money. π - best response of a representative individual on whether to accept money.

- V_j, $j = (0, 1, m)$ - value functions of a producer, commodity trader, money trader.

Flow Bellman Equations

For rate of time preference $r > 0$,

$$\begin{aligned} rV_0 &= \alpha(V_1 - V_0) \\ rV_1 &= \beta(1-\mu)x^2(U - \varepsilon + V_0 - V_1) + \beta\mu x \max_{\pi} \pi(V_m - V_1) \\ rV_m &= \beta(1-\mu)\Pi x(U - \varepsilon + V_0 - V_m) \end{aligned}$$

Program depends on strategies of others and on the proportion of traders holding money, μ, determined endogenously as $\mu(\Pi, M)$.

Steady State Characterization

- Results:
  - Unique μ for each Π, M, and $\mu(0, \Pi) = 0$, $\mu(1, \Pi) = 1$.
  - $\partial\mu/\partial M > 0$
  - $\partial\mu/\partial\Pi > 0$

- Given $\mu(M, \Pi)$, we have the steady state as

$$\begin{aligned} N_0 &= \phi/(\alpha+\phi) \\ N_1 &= (1-\mu)\alpha/(\alpha+\phi) \\ \phi &= \beta(1-\mu)[\mu x\Pi + (1-\mu)x^2]. \end{aligned}$$

Equilibria

- $\mu(M, \Pi)$ is sufficient information for the agent to determine his best-response strategy.

- Three equilibrium possibilities emerge.
  1. If $\Pi < x$, then $V_m < V_1$, and $\pi = 0$ - nonmonetary equilibrium.
  2. If $\Pi > x$, then $V_m > V_1$, and $\pi = 1$ - pure-monetary equilibrium.
  3. If $\Pi = x$, then $V_m = V_1$, and $\pi \in [0, 1]$ - mixed-money equilibrium (symmetric mixed-strategy).

- These equilibria are self-fulfilling.

Welfare

- Compare welfare across different equilibria.

- Let $\alpha \to \infty$ (instantaneous production) - all agents are now traders, and $\mu = M$.

- Using the value functions, compute the payoffs under the simplified assumptions.

- With N, M and P denoting nonmonetary, mixed- and pure-monetary equilibria, we get:
  - $V_1^N = V_1^M < V_1^P$
  - $V_m^N < V_m^M < V_m^P$

Welfare Effects of Changes in Money Supply

- Increase the number of agents endowed with money (hence, reduce the number endowed with real goods).
- In nonmonetary and mixed-monetary equilibrium, all agents are better off the lower the M - money does not help ameliorate trading problems there, so reducing the number of real goods in the economy makes everyone worse off.
- In pure monetary equilibrium:
  - if $x \geq 1/2$, the optimal money supply is $M = 0$. (Barter is not very difficult);
  - for $x < 1/2$, $M > 0$. (Pure barter is sufficiently difficult). $M \to 1/2$ as $x \to 0$.

Specialization

- Let arrival rate in the production process be: $\alpha = \alpha(x)$, $\alpha' < 0$. Specialization: higher productivity, lower customer base.

- Now let agents choose x prior to entering the production process, which in equilibrium gives X, the fraction of output that a given agent will consume. Payoffs given by

$$\begin{aligned} rV_0 &= \alpha(x)(V_1(x) - V_0) \\ rV_1(x) &= \beta(1-\mu)Xx(U - \varepsilon + V_0 - V_1(x)) + \beta\mu x\Pi(V_m - V_1(x)) \\ rV_m &= \beta(1-\mu)\Pi X(U - \varepsilon + V_0 - V_m). \end{aligned}$$

Producer maximizes the RHS of the first equation. Solve for symmetric equilibrium with $X = x(X, \mu, \Pi)$ and $M = M(X, \mu, \Pi)$.

Specialization

- Specialization is lowest (X highest) in the nonmonetary equilibrium, and highest (X lowest) in the pure-monetary equilibrium: $X^P < X^M < X^N$.

- When money circulates, there is the most advantage to specialization: productivity is highest, barter is the most difficult, but money overcomes that.

Effects of Money Supply on Specialization

- Can show that an increase in M increases X (decreases specialization) in the nonmonetary equilibrium, decreases X (increases specialization) in the pure-monetary equilibrium, and has no effect in the mixed-monetary equilibrium.

- An increase in M encourages specialization because more money in the economy means higher marketability of the good. In the nonmonetary equilibrium, more M just drives out real goods (so fewer goods remain to be chased by too much money...)

Effects of Reducing Trading Frictions

- An increase in β produces a reduction in trading frictions: higher probability of matching at a given point in time.
- As β increases, X falls (specialization increases) in the pure monetary equilibrium, since it is easier to meet agents in trade.
- This makes barter difficult ($X \to 0$ as $\beta \to \infty$); ratio of barter to monetary trades vanishes.
- Bottom line: economy settles on the use of money; this encourages specialization, which in turn inhibits barter.