

Generation-2 Monetary Search Models: Trejos and Wright (JPE, 1995)

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Introduction

- First-generation models: endogenize the use of (commodity and fiat) money as a medium of exchange.
- Money and goods essentially indivisible. As a result: goods trade one for one.
- Exogenous pricing: *“While the papers analyze in detail the exchange process, they neglect the determination of exchange rates”*.

Incorporating Pricing

- Use the exchange process modeled in Kiyotaki and Wright (1989, 1991, 1993) and others. Let go of divisibility of goods.
- Pricing mechanism bargaining.
- Two closely related papers:
 - Trejos and Wright (JPE, 1995) incorporate bargaining into the model of bilateral exchange a la KW'93.
 - With slightly different assumptions on the fundamentals, and slightly different results, Shi (JET, 1995) does a similar exercise. Studies somewhat different extensions and applications.

“Search, Bargaining, Money and Prices”, Trejos and Wright 1995

- Continuum of infinitely lived agents, measure 1.
- Goods are nonstorable, *perfectly divisible*.
- Specialization: each agent has a demand for a variety of services that can be produced by share of agents x ; each agent can produce a variety of goods demanded by share x of population.
- Agents cannot (or *sometimes* cannot) produce goods for their own consumption.

(a). Monetary Theory (Exchange Side)

- Fraction $M \in [0, 1]$ of agents endowed with one unit of fiat money: cannot be produced or consumed by agents.
- Money is indivisible: always spend all the cash. (Relaxed later)
→ buyers are a group of measure M , have one unit of money; sellers are of measure $1 - M$, have no money.
- Buyers do not sell to buyers → no one ever gets a second unit of money.
- Barter is ruled out: sellers do not meet, buyers do not produce
- e.g. as in Kiyotaki and Wright 1993. (Relaxed later).

Exchange Side

- If 1 unit of money buys q units of consumption, we have $p = 1/q$.
- Buyer's utility $u(q)$. Seller's disutility $c(q)$. $u' > 0$, $u'' \leq 0$; $c' > 0$, $c'' > 0$. $u(0) = c(0) = 0$.
- Frictions in travel: amount of time it takes to get to a seller is drawn from an exponential distribution. (Equivalent: Poisson arrival process, like before). Arrival rate β .
- Thus: find a seller with probability $\beta(1 - M)$; a buyer with probability βM . With probability x the seller has the wanted good and trade can take place. Let $\beta x = 1$.

A Meeting

- Buyer and seller in an appropriate meeting bargain over q .
- If agree, they trade for money; buyer becomes seller, seller becomes buyer.
- Suppose for now that $q_t = Q_t$ is given exogenously (derive next).
- Consider a short discrete time interval of length $\Delta > 0$.

Value Functions

$$V_{st} = \frac{1}{1+r\Delta} \{M\Delta [V_{bt+\Delta} - c(Q_{t+\Delta})] + (1-M\Delta)V_{st+\Delta} + o(\Delta)\}$$

$$V_{bt} = \frac{1}{1+r\Delta} \{(1-M)\Delta [V_{st+\Delta} + u(Q_{t+\Delta})] + M\Delta V_{bt+\Delta} + o(\Delta)\}$$

Simplify and divide by Δ , then take limit as $\Delta \rightarrow 0$:

$$rV_{st} = M [V_{bt} - V_{st} - c(Q_t)] + \dot{V}_{st}$$

$$rV_{bt} = (1-M) [u(Q_t) + V_{st} - V_{bt}] + \dot{V}_{bt}$$

Value Functions

In steady state, assuming $Q_t = Q$, we get

$$V_s(Q) = \frac{M}{r(1+r)} [(1-M)u(Q) - (r+1-M)c(Q)]$$

$$V_b(Q) = \frac{1-M}{r(1+r)} [(r+M)u(Q) - Mc(Q)]$$

- Observe $V_b - c(Q) > V_s > 0$ iff $(1-M)u(Q) > (1+r-M)c(Q)$
- condition under which sellers voluntarily accept money.
- Then that $V_b > 0$ - money yields has value, even though it yields no direct utility.

(b). Price Theory (Bargaining)

Nash (1950) or Rubinstein (1982) bargaining.

- Now take as given that $V_b > 0$ - money has value.
- Buyer and seller meet: one is chosen at random, with probability $1/2$, to propose a value q .
- The other accepts or rejects. If accepts: trade occurs. If rejects: either can walk away and search for a new partner (no reconvening), or after a wait of Δ , one is chosen at random again to propose a new q .
- No additional arrivals during Δ (relaxed later).
- Unique SPNE, taking V_b and V_s as given: seller proposes $q_s(\Delta)$, buyer proposes $q_b(\Delta)$, and proposals are always accepted at first round.
- As $\Delta \rightarrow 0$, q_b and $q_s \rightarrow q$.

Bargaining

q_b and q_s have to be such that both parties are indifferent between accepting and rejecting:

$$V_s + u(q_s) = \frac{1}{1 + r\Delta} \left[V_s + \frac{1}{2}u(q_s) + \frac{1}{2}u(q_b) \right] \quad (\text{B})$$

$$V_b - c(q_b) = \frac{1}{1 + r\Delta} \left[V_b - \frac{1}{2}c(q_s) - \frac{1}{2}c(q_b) \right] \quad (\text{S})$$

As $\Delta \rightarrow 0$, $q_s, q_b \rightarrow q$.

Bargaining Solution

The limit q can be represented, taking Q as given, as:

$$q = \arg \max [V_s + u(q)][V_b - c(q)]$$

subject to individual rationality constraints

$$V_b(Q) - c(q) \geq V_s(Q) \quad (\text{S})$$

$$V_s(Q) + u(q) \geq V_b(Q) \quad (\text{B})$$

(c) Equilibria

We now bring the two pieces together.

- A steady state equilibrium is a list (Q, V_s, V_b) that satisfies:
 1. $q = Q$ solves the bargaining problem above, taking $V_s(Q)$ and $V_b(Q)$ as given;
 2. V_s and V_b maximize individual lifetime utility, taking Q as given.
- **Unconstrained** equilibrium: $q = Q$ solves the Nash problem, with neither IR constraint binding.
- **Constrained** equilibrium: Q is such that one of the constraints binds, while the solution to the Nash problem violates one of the constraints.
- **Monetary** equilibrium - $Q > 0$; **nonmonetary** equilibrium - $Q = 0$.

Equilibria

- For any $r > 0$ and $M \in (0, 1)$, there exists a nonmonetary steady state equilibrium and a unique monetary steady state equilibrium. The monetary equilibrium is unconstrained, and $u'(q) > c'(q)$.
- This establishes existence (and uniqueness) of equilibrium where fiat money has positive value determined endogenously through bargaining.
- Uniqueness depends crucially on assumption of no-barter: if add barter, equilibria come in pairs.
- Crucial result: $u'(q) > c'(q) \Rightarrow q < q^*$, where q^* satisfies $u'(q) = c'(q)$ and is the social planner solution. That is, the monetary equilibrium is socially inefficient.
- Intuition: this result is driven by discounting.

Effects of Changes in Money Supply

- Increase M , the number of agents with money (here, increasing on the intensive margin would be neutral).
- Can show that $\exists \underline{M}(r) > 0$ for small r , such that $\partial q / \partial M \leq 0$ iff $M \geq \underline{M}$.
- When M is small, an increase in money supply along extensive margin may increase q - i.e. decrease the price. “Liquidity effect” - this is where money helps overcome search frictions, increasing number of meetings.
- For $M \geq \underline{M}$, as M increases, this has an inflationary effect - q falls, p rises.
- Formally: in a model with divisible commodities, the optimal (welfare-maximizing) M balances the trade-off between providing liquidity to agents and increasing the price level.

(d) Extension 1: Exogenous Breakdowns

- Bargaining game changes: now if an offer is rejected, in the intervening Δ , other trading partners may arrive, at same rates $(\beta x)M$ and $(\beta x)(1 - M)$; if a suitable match, assume that the person leaves the previous match.
- This increases the threat of delay: possible to be left without a partner if an offer is rejected.
- Here the unique SPNE is still that offers are always accepted at first round, and q_s and q_b converge to q as $\Delta \rightarrow 0$. Now can show that q satisfies

$$\max(V_s + u(q) - V_b)(V_b - c(q) + V_s)$$

Equilibria with Exogenous Breakdowns

- Still have a nonmonetary steady state and a unique *unconstrained* monetary steady state, but may not have $u'(q) > c'(q)$.
- Recall from before, we had $q < q^*$ because of discounting. Now it is complicated by the positive probability of match breakdown, which affects relative bargaining power.
- If M is small, there are few buyers. Then the breakdown threat is more heavily on sellers (buyers' relative bargaining power increases), so q goes up. Could have $q > q^*$. If M is large, the opposite is the case.
- Can show that at $M = 1/2$, $q \rightarrow q^*$ as $r \rightarrow 0$.

(e) Extension 2: Allow Barter

- Allow sellers to meet other sellers; will barter with probability $\beta(1 - M)x^2$.
- In barter meetings, $q = q^*$.
- For money meetings can show:
 - With no exogenous breakdowns, there exists $\tilde{r}(x, M) > 0$ such that $r > \tilde{r} \Rightarrow$ no monetary equilibria; otherwise, there are two: one constrained and one unconstrained.
 - With exogenous breakdowns, there exists $\hat{r}(x, M) > 0$ such that $r > \hat{r} \Rightarrow$ no monetary equilibria; otherwise, there are generically an even number of monetary equilibria, all unconstrained.
- Notice that now, to have monetary equilibria, we need agents to be patient enough.

(f) Extension 3: Divisible Money

- Suppose agents can hold any $m \in \mathbb{R}_+$. $F(m)$ - distribution of money holdings across agents; $V(m)$ - value function of agent.
- No barter, Poisson arrival of meetings with rate β , buy or sell with probability x . Let $d(m_s, m_b)$ and $q(m_s, m_b)$ be the quantity of goods and money exchanged in a trade meeting.
- $d(\cdot)$ and $q(\cdot)$ solve the Nash problem

$$\begin{aligned} \max_{q,d} & [u(q) + V(m_b - d)][-c(q) + V(m_s + d)] \\ \text{s.t.} & \quad u(q) + V(m_b - d) \geq V(m_b) \\ & \quad -c(q) + V(m_s + d) \geq V(m_s) \end{aligned}$$

- Given $M = \int m dF(m)$, the equilibrium definition now includes F which has to be a stationary distribution of money holdings consistent with the bargaining solution and the individual V .

Equilibrium with Divisible Money

- Problem: nontrivial distribution of money holdings. Cannot prove existence or completely characterize equilibria analytically.
- Can show:
 - $q(m_b, m_s) < q^*$ for all m_b, m_s ;
 - q depends only on $m_s + m_b$, not on individual money holdings in a given meeting;
 - d depends on m_s, m_b individually;
 - richer agents (with more money) get less money when they sell and pay more when they buy.