Liquidity

Irina A. Telyukova

UCSD ECON 213

Advanced Monetary Theory

Spring 2008

Introduction

- Last time we encountered Lipman and McCall (1986) and their definitions of liquidity.

- Search models are the natural setting, they say, within which to study liquidity of assets - connected to notions such as uncertain arrival times, uncertain offers, etc.

- We pursue liquidity within modern monetary search models: Lagos (2008) “Asset Prices and Liquidity in an Exchange Economy”.

Introduction

- Idea: to develop a model in which assets are valued not only as claims to consumption but also for their *liquidity* - the degree to which they are useful in facilitating exchange.
- Decentralized trade and exchange motive (desire to trade) facilitate the need for a medium of exchange.
- An economy with an equity share and a one-period government-issued risk-free real bill. The bill may or may not have legal advantages as a medium of exchange (consider both possibilities), but agents are always free to choose which asset to use as medium of exchange.
- Model accounts for the risk-free puzzle: low rate of return on bonds; as well as the equity premium puzzle: agents hold relatively little equity, despite its having higher returns (would benefit from disinvesting in bonds in favor of equity).

Environment

- $[0, 1]$ continuum of agents. Infinite horizon. Discrete time.
- Two subperiods in each period: DM followed by CM.
- Three types of nonstorable goods:
  - general y_t - consumption goods, produced/traded in CM
  - "apples" c_t - dividend and consumption goods, fruit of Lucas trees, consumed in CM
  - "coconuts" Q_t - endowment goods, traded/consumed in DM
- Durables: Lucas trees, number = number of agents. Yield dividend (apples) in CM every period.
- Assets: each tree has outstanding one share, perfectly divisible: dividends payable to bearer. Also: one-period risk-free government (real) bill. Each pays off 1 unit of dividend (apple) at maturity.

Shocks

- Aggregate productivity shock: Z_t, $Z_{t+1} = x_{t+1} Z_t$, $x_{t+1} = \{\gamma_1, ..., \gamma_n\}$ with Markov transition probabilities μ_{ij}. x_t realizes at beginning of t.
  - Dividend is $d_t = Z_t$
  - In CM, each agent is endowed with $\bar{n}$ units of time which he can convert into Z_t units of general goods.

- Idiosyncratic shock, tied to aggregate productivity shock: in DM (at beginning of t), 1/2 agents, randomly chosen, are endowed with $(1+\varepsilon)\kappa Z_t$, 1/2 with $(1-\varepsilon)\kappa Z_t$. κZ_t - total endowment goods.

Decentralized Market

- Preferences are

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(Q_t) + U(c_t) + v(y_t) - A_t h_t \right] \right\}$$

- Subperiod 1: DM. Shock x_t realizes, endowment shock realizes. Agents trade financial assets and endowment goods in bilateral exchange. Terms of trade determined by bargaining. Low-endowment agents become buyers; high-endowment - sellers. No double-coincidence.

- $\hat{\alpha}$ - probability of a meeting. $\hat{\alpha}/2$ - probability of meeting agent with opposite endowment position (single-coincidence/trade meeting).

- No contracting/trading histories private - no credit. Need for a medium of exchange. Big question: which asset arises as such?

Centralized Market; Assets

- Subperiod 2: CM. Agents trade dividend and general goods, financial assets and labor.

- B_t - total stock of government bills, redeemed before end of CM.

- Government sells bills B_{t+1} in CM: $B_t = \phi_t^b B_{t+1} + \tau_t$. ϕ_t^b

- Assets are perfectly recognizable, cannot be forged, and can be traded in both CM and DM.

- At $t = 0$, agents endowed with a_0^s shares and a_0^b bonds.

Liquidity

- With probability $\theta_2 \in [0, 1]$, agents encounter meetings where either asset can be traded in DM. With $\theta_1 = 1 - \theta_2$, only bonds can be traded in DM.

- $\theta_1 = 0$ - no restriction in trade of either asset at any time.

- θ_2 measures the degree of liquidity (θ_1 - illiquidity) of equity shares.

Value Functions

- $\mathbf{a} = (a_t^b, a_t^s)$ be the agent's portfolio.

- Aggregate state of the economy: $\mathbf{s}_t = (d_t, x_t, B_t)$ - realization of dividend process (Z_t), aggregate shock, total stock of bonds.

- $V_j(\mathbf{a}_t, \mathbf{s}_t)$, $j \in \{h, l\}$ - value of agent in DM with high/low endowment. $W(\mathbf{a}_t, \mathbf{s}_t)$ - value of agent in CM.

- Buyers with assets $\mathbf{a}$ and seller with $\tilde{\mathbf{a}}$ in meeting $i \in \{1, 2\}$ bargain over $(q^i(\mathbf{a}, \tilde{\mathbf{a}}), p^i(\mathbf{a}, \tilde{\mathbf{a}}))$, with $\mathbf{p}$ denoting portfolio of assets used by buyer.

Decentralized Market Values

$$
\begin{aligned}
V(\mathbf{a},\mathbf{s}) &= \frac{1}{2}\left[V_l(\mathbf{a},\mathbf{s})+V_h(\mathbf{a},\mathbf{s})\right] \\
V_h(\mathbf{a},\mathbf{s}) &= \frac{\hat{\alpha}}{2}\sum_{i=1,2}\theta_i\int\left\{u[(1+\varepsilon)\kappa d-q^i(\tilde{\mathbf{a}},\mathbf{a})]+W[\mathbf{a}+\mathbf{p}^i(\tilde{\mathbf{a}},\mathbf{a}),\mathbf{s}]\right\}d\mathbf{G}(\tilde{\mathbf{a}}) \\
&+ (1-\frac{\hat{\alpha}}{2})\left\{u[(1+\varepsilon)\kappa d]+W(\mathbf{a},\mathbf{s})\right\} \\
V_l(\mathbf{a},\mathbf{s}) &= \frac{\hat{\alpha}}{2}\sum_{i=1,2}\theta_i\int\left\{u[(1-\varepsilon)\kappa d+q^i(\mathbf{a},\tilde{\mathbf{a}})]+W[\mathbf{a}-\mathbf{p}^i(\mathbf{a},\tilde{\mathbf{a}}),\mathbf{s}]\right\}d\mathbf{G}(\tilde{\mathbf{a}}) \\
&+ (1-\frac{\hat{\alpha}}{2})\left\{u[(1-\varepsilon)\kappa d]+W(\mathbf{a},\mathbf{s})\right\}
\end{aligned}
$$

Centralized Market Value

$$\begin{aligned} W(\mathbf{a},\mathbf{s}) &= \max_{c,y,n,h,\mathbf{a}'} \{U(c) + v(y) - Ah + \beta V(\mathbf{a}',\mathbf{s}')\} \\ \text{s.t. } c + wn + \phi\mathbf{a}' &= (\phi^s + d)a^s + a^b + wh - \tau \\ y &= Zn \\ h &\in [0,\bar{n}] \\ n &\geq 0 \end{aligned}$$

Bargaining

Consider the case of Nash bargaining in a meeting of type $i \in \{1,2\}$ with buyer-take-all (can generalize).

$$\max_{q^i,\mathbf{p}^i}\{u[(1-\varepsilon)\kappa d + q^i] + W(\mathbf{a}-\mathbf{p}^i,\mathbf{s}) - u[(1-\varepsilon)\kappa d] - W(\mathbf{a},\mathbf{s})\}$$

$$\text{s.t. } u[(1+\varepsilon)\kappa d - q^i] + W(\tilde{\mathbf{a}}+\mathbf{p}^i,\mathbf{s}) - u[(1+\varepsilon)\kappa d] - W(\tilde{\mathbf{a}},\mathbf{s}) \geq 0$$

$$\mathbf{p}^1 = (p^b, 0) \text{ and } p^b \leq a^b$$

$$\mathbf{p}^2 = (p^b, p^s) \leq \mathbf{a}$$

Bargaining Solution

Let $\lambda^1 = \{\lambda^b, 0\}$; $\lambda^1 = \{\lambda^b, \lambda^s\} = \{A/w, (\phi^s + d)\lambda^b\}$.

$$q^i(\mathbf{a}, \tilde{\mathbf{a}}) = \begin{cases} \varepsilon\kappa d & \text{if } \lambda^i \mathbf{a} \geq u[(1+\varepsilon)\kappa d] - u[\kappa d] \\ \hat{q}(\mathbf{a}, \tilde{\mathbf{a}}) & \text{otherwise} \end{cases}$$

where $\hat{q}$ solves

$$u[(1+\varepsilon)\kappa d] - u[(1+\varepsilon)\kappa d - q] = \lambda^i \mathbf{a}$$

Equilibrium

Given $\{d_t\}$ and $\{B_t, \tau_t\}$, an equilibrium is an allocation $\{c_t, y_t, n_t, h_t, \mathbf{a}_{t+1}\}_{t=0}^{\infty}$, prices $\{w_t, \phi_t\}_{t=0}^{\infty}$ and bilateral terms of trade $\{q^i\}_{t=0}^{\infty}$, such that:

- individual choices $\{c_t, y_t, n_t, h_t, \mathbf{a}_{t+1}\}$ solve the CM problem of the agent given prices;
- terms of trade are determined by Nash bargaining;
- prices clear the markets;
- government budget constraint is satisfied.

Returns on Assets

In equilibrium, asset prices satisfy

$$\begin{aligned} U'(d)\phi^s &= \beta \mathbb{E} U'(d') L'^s (\phi'^s + d') \\ U'(d)\phi^b &= \beta \mathbb{E} U'(d') L'^b \end{aligned}$$

$$\begin{aligned} L^s &= 1 + \frac{\hat{\alpha}}{4}(1-\theta_1)\left\{\frac{u'[(1-\varepsilon)\kappa d + q^2]}{u'[(1+\varepsilon)\kappa d - q^2]} - 1\right\} \\ L^b &= L^s + \frac{\hat{\alpha}}{4}\theta_1\left\{\frac{u'[(1-\varepsilon)\kappa d + q^1]}{u'[(1+\varepsilon)\kappa d - q^1]} - 1\right\} \end{aligned}$$

L^j are liquidity returns and are > 1 if the feasibility constraint $\mathbf{p} \leq \mathbf{a}$ is binding.

Comparing Returns

- Measured returns on assets:

$$\hat{R}^s = \frac{\phi^s + d}{\phi^s_{-1}}$$
$$\hat{R}^b = \frac{1}{\phi^b_{-1}}$$

- Full (liquidity-augmented) returns on assets:

$$R^s = L^s \hat{R}^s$$
$$R^b = L^b \hat{R}^b$$

Euler Equations

- Using full returns:

$$\mathbb{E}\left\{\beta\frac{U'(d')}{U'(d)}(R'^s - R'^b)\right\} = 0$$
$$\mathbb{E}\left\{\beta\frac{U'(d')}{U'(d)}(R'^b - 1)\right\} = 0$$

- Using measured returns:

$$\mathbb{E}\left\{\beta\frac{U'(d')}{U'(d)}(\hat{R}'^s - \hat{R}'^b)\right\} = \mathbb{E}\left\{\beta\frac{U'(d')}{U'(d)}((L'^b - 1)\hat{R}'^b - (L'^s - 1)\hat{R}'^s)\right\}$$
$$\mathbb{E}\left\{\beta\frac{U'(d')}{U'(d)}(\hat{R}'^b - 1)\right\} = -\mathbb{E}\left\{\beta\frac{U'(d')}{U'(d)}(L'^b - 1)\hat{R}'^b\right\}$$

- If $L^s = L^b = 1$, the economy has no liquidity needs, and measured and full returns are equal to each other on both assets. Call RHS's ω^e and ω^b. Predictions: $\omega^e = \omega^b = 0$.

In the Data

- Typically, one would estimate the RHS of the equations above using measured returns, and find that the prediction $\omega^e = \omega^b = 0$ is violated.

- In fact, one finds $\omega^e > 0$ and $\omega^b < 0$: equity premium puzzle and risk-free-rate puzzle.

- This model accounts for both if we use full, rather than measured, returns for pricing assets.

- Outstanding question: why are some assets more generally accepted as media of exchange than other assets? Which fundamental features lead to differences in acceptability?