"Dynamic Taxation, Private Information and Money" by Chris Waller (2007)

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Introduction

- Friedman: money should not be taxed via inflation, not to distort its rate of return relative to other assets.
- Why do we not see the Friedman rule in practice? Phelps (1973): in the second-best world, governments must use distortionary taxation on goods to finance spending on transfers. So distortions should be equated: governments should tax money via inflation as well.
- Idea: to examine optimality of the inflation tax in dynamic general equilibrium models where governments must resort to distortionary taxation to finance spending.
- Existing approaches:
  - Ramsey approach (no lump-sum tax allowed);
  - reduced-form justification for money.

Introduction

- Would like to have:
  - the government is free to use lump-sum taxes, and decides whether to use them;
  - frictions that give rise to money as a medium of exchange are explicitly modeled.

- Merge dynamic public finance literature (dynamic Mirrleesian models) with monetary search literature.

- Model: three-period OLG model. Agents subject to preference shocks which are private information.

Record-Keeping

- Study the insight that money is a form of record-keeping (Kocherlakota, 1998).

- Three versions of the model:
  1. Social planner, no money, no record-keeping technology (autarky).
  2. Social planner, no money, with record keeping: planner assigns consumption to agents based on reports of individual shocks -
     - shocks are public information (first-best);
     - shocks are private information (constrained optimum).
  3. Decentralized, with money, no record keeping.

Questions

1. Can the constrained planner allocation be implemented with the use of fiat money and optimally chosen fiscal policy?

2. Does the optimal fiscal policy require the use of inflation taxation even if lump-sum taxes are available?

Environment

- A continuum [0,1] of agents is born every other period.
- At birth, young (Y) agents meet old (O) agents. This is CM.
- Young agents are endowed with labor, convert 1 unit to 1 unit of perishable goods.
- Old agents don't have labor, must trade with young to consume.
- Middle-aged (M) agents trade with other middle-aged agents in the DM.
- Discount between periods at rates β_1 and β_2.

The Middle Ages

- Agents are subject to idiosyncratic preference shocks.

- Preferences in DM: $\varepsilon_b u(q) - \varepsilon_s \psi(q)$.

- With probability $\sigma \leq 1/2$, $\varepsilon_b = 1$, $\varepsilon_s = 0$; with probability σ, $\varepsilon_b = 0$, $\varepsilon_s = 1$. $1 - 2\sigma$ - probability of being idle.

Lifetime Utility

An agent born at time $t-1$ has lifetime utility

$$W_{y,t-1} = U(C_{y,t-1}) - v(h_{t-1}) + \beta_1 V_t$$

where

$$V_t = \sigma[u(q_t^b) - \psi(q_t^s)] + \sigma\beta_2[U(C^b_{o,t+1}) + U(C^s_{o,t+1})] + (1-2\sigma)\beta_2 U(C^n_{o,t+1})$$

- Superscripts - buyer, seller, nontrader.
- Subscripts - young, old.

I. No Money, No Record-Keeping

- Agents will not be able to trade in DM and the young will not supply the old with consumption - autarky.

- $C_y = h$, $q^b = q^s = C_o^j = 0$, $j \in \{b, s, n\}$.

- Lifetime welfare is given by $W^a = U(C_y^a) - v(C_y^a) + \beta_1 \beta_2 U(0)$ with $U'(C_y) = v'(C_y)$.

II-a. No Money, Record-Keeping, Public Information

- Agents send a report to the planner about the realization of their idiosyncratic preference shock in the DM.
- Planner has a technology that allows her to keep track of agents' reports.
- Conditional on the report, the planner gives the agent a sequence of consumption in the middle and old age (DM and CM).
- If report is "buyer", agent is given q^b at t and C_o^b at $t+1$. If "seller", agent delivers q^s and is given C_o^s. If nontrader, then he receives C_o^n.
- The planner can commit to the sequence of consumption.

Planner Problem

Suppose economy begins in $t = -1$ with an initial generation of the middle-aged. The planner chooses CM allocations $\{C_y, h, C_o^j\} = \{C_{y,2t}, h_{2t}, C^b_{o,2t}, C^s_{o,2t}, C^n_{o,2t}\}_{t=0}^{\infty}$ and DM allocations $\{q^b, q^s\} = \{q^b_{2t+1}, q^s_{2t+1}\}_{t=0}^{\infty}$ to maximize

$$
\begin{aligned}
\mathcal{W} \quad = \quad & \sigma\lambda^{-1}[u(q^b_{-1}) - \psi(q^s_{-1})] + \beta_2\lambda^{-1}[\sigma U(C^b_{o,0}) + \sigma U(C^s_{o,0}) + (1-2\sigma)U(C^n_{o,0})] \\
+ \quad & \sum_{t=0}^{\infty} \lambda^t \{U(C_{y,2t}) - v(h_{2t}) + \sigma\beta_1[u(q^b_{2t+1}) - \psi(q^s_{2t+1})]\} \\
+ \quad & \sum_{t=0}^{\infty} \lambda^t \beta_1\beta_2[\sigma U(C^b_{o,2t+2}) + \sigma U(C^s_{o,2t+2}) + (1-2\sigma)U(C^n_{o,2t+2})]
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t. } h_{2t} \quad & \geq \quad \sigma U(C^b_{o,2t}) + \sigma U(C^s_{o,2t}) + (1-2\sigma)U(C^n_{o,2t}) + C_{y,2t} \forall t \\
\sigma q^s_{2t+1} \quad & \geq \quad \sigma q^b_{2t+1} \forall t
\end{aligned}
$$

λ^t - Pareto weights assigned to generation born at $2t$.

Characterization

If planner can assign production and consumption to the agents, the planner allocation - unconstrained optimum (no informational frictions) - is given by

$$\begin{aligned}
u'(q^b_{2t+1}) &= \psi'(q^s_{2t+1})\forall t \geq 0 \\
U'(C_{y,2t}) &= v'(h_{2t})\forall t \geq 0 \\
U'(C^b_{o,2t}) &= U'(C^s_{o,2t}) = U'(C^n_{o,2t})\forall t \geq 0 \\
U'(C_{y,0}) &= \beta_2 \lambda^{-1} U'(C^j_{o,0})\forall j \\
U'(C_{y,2t}) &= \beta_1 \beta_2 \lambda^{-1} U'(C^j_{o,2t})\forall t > 0 \\
U'(C_{y,2t}) &= \frac{v'(h_{2t})}{v'(h_{2t+2})}\beta_1 \beta_2 \mathbb{E} U'(C^j_{o,2t+2})\forall j
\end{aligned}$$

All consumption and production is efficient and marginal utilities are equalized intra- and intertemporally, subject to weights being equal across generations. This is first best.

II-b. No Money, Record Keeping, Private Information

- The above allocation cannot be implemented if shocks are private information.
- Reason: everyone in old age gets the same consumption regardless of DM status - no incentive to produce (to reveal seller status truthfully), as there is no reward for production. Can do better by lying and not expending the effort.
- Even if planner can force quantities on buyers and sellers, this is useless if planner cannot observe who is who.
- We are looking for incentive-compatible equilibria that induce truthful revelation.

Incentive Compatibility Constraints

The planner will now solve the same problem as before, but with these additional constraints (the ones that may bind only):

- Buyer does not misreport as nontrader -

$$u(q^b_{2t+1}) + \beta_2 U(C^b_{o,2t+2}) - \beta_2 U(C^n_{o,2t+2}) \geq 0$$

- Seller does not misreport as nontrader -

$$-\psi(q^s_{2t+1}) + \beta_2 U(C^s_{o,2t+2}) - \beta_2 U(C^n_{o,2t+2}) \geq 0$$

- Nontrader does not misreport as buyer -

$$U(C^n_{o,2t+2}) - U(C^b_{o,2t+2}) \geq 0$$

- The young produce rather than going into autarky -

$$U(C_{y,t}) - v(h_t) + \beta_2 \sigma [u(q^b_{2t+1}) - c(q^s_{2t+1})] + \beta_1 \beta_2 \mathbb{E} U(C^o_{2t+2}) \geq W^a$$

Characterization

Constrained optimum - planner trades off efficiency and incentive-compatible risk-sharing. Given by:

$$\begin{aligned}
\psi(q^s_{2t+1}) &= \beta_2 U(C^s_{o,2t+2}) - \beta_2 U(C^n_{o,2t+2}) \forall t \geq -1 \\
\frac{u'(q^b_{2t+1})}{\psi'(q^s_{2t+1})} &= \frac{U(C^b_{o,2t+2})}{\sigma U(C^b_{o,2t+2}) + (1-\sigma) U(C^s_{o,2t+2})} \forall t \geq -1 \\
U'(C_{y,2t}) &= v'(h_{2t}) \forall t \geq 0 \\
C^b_{o,2t+2} &= C^n_{o,2t+2} < C^s_{o,2t+2} \forall t \geq 0 \\
U'(C_{y,0}) &= \beta_2 \lambda^{-1} \left[\sigma \frac{1}{U'(C^s_{o,0})} + \sigma \frac{1}{U'(C^b_{o,0})} + (1-2\sigma) \frac{1}{U'(C^n_{o,0})} \right]^{-1} \\
U'(C_{y,2t+2}) &= \beta_1 \beta_2 \lambda^{-1} \left[\sigma \frac{1}{U'(C^s_{o,2t+2})} + \sigma \frac{1}{U'(C^b_{o,2t+2})} + (1-2\sigma) \frac{1}{U'(C^n_{o,2t+2})} \right]^{-1} \\
U'(C_{y,2t}) &= \frac{v'(h_{2t})}{v'(h_{2t+2})} \beta_1 \beta_2 \left[\mathbb{E} \frac{1}{U'(C_{o,2t+2})} \right]^{-1}
\end{aligned}$$

III. Money, No Record Keeping

- Maintain private information assumption.
- Money - the only durable asset in the economy. Divisible, and no bound on holdings.
- Government receives no reports. Instead, gives agents fiat money.
- Money is injected to middle-aged agents (in CM, plays no role). $M_{t+2} = \gamma_t M_t$, where $\gamma_t = 1 + \pi_t$ is rate of growth of money from t until $t+2$.

Markets

- CM - firms hire labor, linear production technology. Market is perfectly competitive. $\phi = 1/p$. Gross rate of return on money is $R_m = \phi_{+1}/\phi_{-1}$.

- DM - preference shocks lead to double-coincidence problem. Sellers enter old age with more money than buyers. But death keeps distribution of money degenerate. (Assume that upon death, the government will cease any remaining balances and redistribute them back to the middle ages.)

- Random matching and bargaining in DM.

- Old agents use cash for consumption.

Role of Money

- Money induces truthful revelation.
- Seller is willing to reveal his type if buyer has money, in exchange for greater consumption in old age.
- Nontraders do not mimic buyers since that means they give up money today for consumption they do not value, and forgo consumption tomorrow that they do value.
- NB: money replicates record-keeping by the planner by inducing truthful revelation on all sides; provides means of consumption in future periods.
- Can money replicate the constrained optimum, if the government can also tax? First characterize agents' problem, then solve the government's problem.

Taxation

- Suppose the government can observe the age of agents, labor supply of the young, and consumption.
- Can impose lump-sum taxes/transfers by age; distortionary labor taxes on the young; non-linear consumption tax on market goods consumed. Consumption tax = tax on real monetary wealth.
- Government budget constraint:

$$T_o = \tau^h h + T_y + \tau^c C_y + \int \eta(C_o^j - T_o) dF(C_o^j)$$

- T_y - lump-sum tax on young; T_o - lump-sum transfers to the old; τ^c - consumption tax on the young; τ^h - labor tax on the young; η - consumption tax on the old.

Centralized Market

- Young agents solve

$$\max_{C_{y,-1},h_{-1},m} \quad U(C_y,-1) - v(h_{-1}) + \beta_1 V_t(m)$$
$$\text{s.t. } (1+\tau^c)C_{y,-1} + \phi_{-1}m = (1-\tau^h)h_{-1} + T_y$$

Characterization:

$$U'(C_{y,-1}) = \frac{1+\tau^c}{1-\tau^h}v'(h_{-1})$$
$$U'(C_{y,-1}) = V'(m)$$

- Old agents: consume and pay taxes using their money, then die.

$$\phi_{-1}m^b_{-1} = C^b_{o,-1} - T_o + \eta(C^b_{o,-1} - T_o)$$
$$\phi_{-1}m^s_{-1} = C^s_{o,-1} - T_o + \eta(C^s_{o,-1} - T_o)$$
$$\phi_{-1}m^n_{-1} = C^n_{o,-1} - T_o + \eta(C^n_{o,-1} - T_o)$$

Decentralized Market

$$\begin{aligned} V_t(m) &= \sigma \int \{u[q^b(m,m^s)] + \beta U[C^b_{o,+1}(m,m^s)]\} dF(m^s) \\ &+ \sigma \int \{-\psi[q^s(m^b,m)] + \beta U[C^s_{o,+1}(m^b,m)]\} dF(m^b) \\ &+ \beta_2(1-2\sigma)U[C^n_{o,+1}(m)] \end{aligned}$$

Buyers and sellers bargain: assume buyer-take-all.

$$\max_{q,d} u(q) + \beta_2 U(C^b_{o,+1}) - \beta_2 U(\hat{C}^b_{o,+1})$$

$$\text{s.t. } d \le m^b$$

$$-\psi(q) + \beta_2 U(C^s_{o,+1}) - \beta_2 U(\hat{C}^s_{o,+1}) \ge 0$$

If there are no distortionary consumption taxes in CM, $u'(q) > \psi'(q)$ as usual in these models - DM consumption is inefficiently low.

Equilibrium

With a non-binding cash constraint, steady-state equilibrium is a list $\{z \equiv \phi M, q, \delta \equiv \phi_{+1} d, C_o^b, C_o^s, C_o^n, C_y, h\}$, such that

$$\begin{aligned}
C_o^s &= z + \delta - \eta(C_o^s - T_o) + T_o \\
C_o^b &= z - \delta - \eta(C_o^b - T_o) + T_o \\
C_o^n &= z - \eta(C_o^n - T_o) + T_o \\
h &= C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^o \\
U'(C_y) &= \frac{1+\tau^c}{1+\tau^h} v'(h) \\
\psi(q) &= \beta_2 [U(C_o^s) - U(C_o^n)] \\
\frac{u'(q)}{\psi'(q)} &= \frac{U'(C_o^b)(1 + \eta'(C_o^s - T_o))}{U'(C_o^s)(1 + \eta'(C_o^b - T_o))} \\
\frac{1}{1+\tau^c} U'(C_y) &= R_m \beta_2 \left[\sigma \frac{U'(C_o^b)}{1 + \eta'(C_o^b - T_o)} + (1-\sigma) \frac{U'(C_o^n)}{1 + \eta'(C_o^n - T_o)} \right]
\end{aligned}$$

Similar for the binding case, with $z = \delta$.

Replicating Constrained Equilibrium

- Assuming a constrained eq'm in the record-keeping economy exists (hard to prove in general), can it be decentralized in the monetary economy using a tax system?
- Idea: analyzing characterizations of equilibrium above to see if it can be brought close to the constrained-optimum case in the no-money setup.
- With non-linear consumption taxes available (as well as lump-sum), it is **possible** to decentralize the constrained optimum, with consumption taxes on the old ranking as $\eta^n > \eta^b > \eta^s$. There is some progressivity and some regressivity in the tax system. Also, rich agents in DM get taxes least heavily - to induce truthful revelation.
- With linear consumption taxes, even with lump-sum taxes also available, cannot decentralize the constrained optimum.

Inflation Tax

- With a full set of taxes available (non-linear), the government may or may not use inflation taxation - depends on the return on money (which can be > 1, $= 1$, < 1). General idea is that the optimal policy is to reduce the after-tax return on money, to reduce consumption risk on the old.

- However, do not want to make return on money too low, else sellers would not reveal truthfully.

- With linear taxes (and lump-sum), can show in an example that if the lump-sum taxes are not too large, it is optimal for the government to resort to the inflation tax.