Econ 220B
James Hamilton

Final Exam
Winter 2003

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 250 points are possible on this exam

1.) (80 points total) This question explores the consequences of measurement error in the explanatory variables. Consider a cross-section of households at a given date. Let C_i denote the measured consumption spending by household i and Y_i the measured income of household i. According to Milton Friedman's permanent income hypothesis, these differ from theoretical constructs called permanent consumption C_i^* and permanent income Y_i^* with measurement errors a_{ci} and a_{yi}: $C_i = C_i^* + a_{ci}$ and $Y_i = Y_i^* + a_{yi}$. Friedman assumed that the measurement errors have mean zero ($E(a_{ci}) = 0$, $E(a_{yi}) = 0$), are uncorrelated with each other ($E(a_{ci}a_{yi}) = 0$), and uncorrelated with either of the permanent values ($E(C_i^* a_{ci}) = 0$, $E(Y_i^* a_{yi}) = 0$, $E(C_i^* a_{yi}) = 0$, $E(Y_i^* a_{ci}) = 0$). Friedman's permanent income hypothesis held that $C_i^* = \beta Y_i^*$. Assume that all variables are stationary and ergodic.

a.) (10 points) The econometrician does not observe the true values C_i^* and Y_i^*. Consider therefore a regression of measured consumption on measured income: $C_i = \beta Y_i + \varepsilon_i$. Use the above assumptions to calculate an expression for ε_i in terms of a_{ci} and a_{yi}.

b.) (20 points) Define what is meant by the plim of an estimator. Calculate the plim of the OLS estimate b and show that it is strictly less than the true value of β.

c.) (10 points) Explain intuitively the source of this downward bias in terms of the correlation between the population regression residual ε_i appearing in part (a) and the explanatory variable Y_i.

d.) (20 points) Friedman suggested that for this model, one could use a constant term ($x_i = 1$ for all i) as an instrument for Y_i. Show that this instrument is (i) relevant and (ii) valid.

e.) (10 points) Using this instrument, show that the two-stage least squares estimator $\hat{\beta}_{2SLS}$ turns out to be $\overline{C}/\overline{Y}$ where $\overline{C} = n^{-1}\sum_{i=1}^{n} C_i$ is the average measured consumption in the sample and $\overline{Y} = n^{-1}\sum_{i=1}^{n} Y_i$ is the average measured income.

f.) (10 points) Calculate the plim of $\hat{\beta}_{2SLS}$.

Econ 220B
James Hamilton

2.) (85 points total) If $E[\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)] = \mathbf{0}$, the GMM estimate of $\boldsymbol{\theta}$ is defined as the value that minimizes

$$T[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]'\hat{\mathbf{S}}^{-1}[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]$$

where

$$\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1} \sum_{t=1}^{T} \mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

and $\hat{\mathbf{S}}$ is a consistent estimate of

$$\mathbf{S} = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} \sum_{v=-\infty}^{\infty} E\left\{[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)]\left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v})\right]'\right\}.$$

The usual asymptotic distribution of the GMM estimator is

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

where $\mathbf{V}$ can be estimated consistently from

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}}\hat{\mathbf{S}}^{-1}\hat{\mathbf{D}}')^{-1}$$

$$\hat{\mathbf{D}}' = \left.\frac{\partial \mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}.$$

a.) (20 points) Consider the regression model $y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$ where $\mathbf{x}_t$ is a $(k \times 1)$ vector of observations on the explanatory variables satisfying $E(\varepsilon_t \mathbf{x}_t) = \mathbf{0}$ and $E(\varepsilon_t \varepsilon_{t-v} \mathbf{x}_t \mathbf{x}_{t-v}') = \sigma^2 \rho^v E(\mathbf{x}_t \mathbf{x}_{t-v}')$ where ρ is a known constant satisfying $0 < \rho < 1$. Use just the first of these conditions $E(\varepsilon_t \mathbf{x}_t) = \mathbf{0}$ to calculate the $(k \times 1)$ vector $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$ and the $(k \times 1)$ vector $\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1}\sum_{t=1}^{T} \mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$. Use this value of $\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)$ to find the value of the GMM estimate $\hat{\boldsymbol{\beta}}$ for this example.

b.) (15 points) For this example, what is the relation between the GMM estimate, the OLS estimate, 2SLS estimate, and the GLS estimate of $\boldsymbol{\beta}$?

c.) (15 points) Use the expression for $\hat{\mathbf{V}}$ above to calculate the asymptotic variance of the GMM estimate $\hat{\boldsymbol{\beta}}$.

d.) (25 points) Suppose you wanted to test the null hypothesis $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ where $\mathbf{R}$ is a known $(m \times k)$ matrix and $\mathbf{r}$ is a known $(m \times 1)$ vector. Use your results from part (c) to calculate the Wald test of this hypothesis and find its asymptotic distribution. How is the statistic you calculated related to the usual OLS F-test of this null hypothesis?

e.) (10 points) Suppose instead you wanted to test the null hypothesis $\boldsymbol{\lambda}(\boldsymbol{\beta}) = \mathbf{0}$ where $\boldsymbol{\lambda} : \Re^k \to \Re^m$ is a known function representing m separate differentiable restrictions. How would you calculate the Wald test statistic for this case?

Econ 220B
James Hamilton

3.) (85 points) Suppose you have an i.i.d. sample of observations y_t drawn from the density $f(y_t;\lambda) = \lambda \exp(-\lambda y_t)$ for $y_t \geq 0$. Recall that the mean of an exponential variable is given by $E(y_t) = 1/\lambda$ and variance is $E(y_t - \lambda^{-1})^2 = \lambda^{-2}$.

a.) (15 points) Calculate the log likelihood function.

b.) (10 points) Calculate the maximum likelihood estimate $\hat{\lambda}$.

c.) (20 points) If you wanted to view maximum likelihood estimation for this example as a special case of GMM, what corresponds to the function $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$? (Refer to question (2) for a review of the GMM formulas, if you like). Prove that $E[\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)] = \mathbf{0}$ for this example.

d.) (20 points) Find the value of $\mathbf{S}$ and $\hat{\mathbf{D}}'$ for this example (again refer to question (2) for notation).

e.) (20 points) Use the expressions you found in part (d) to calculate an estimate of the asymptotic variance of $\hat{\lambda}$. Use your formula to calculate a standard error for the estimate $\hat{\lambda}$.