Econ 220B
James Hamilton

Final Exam
Winter 2005

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 250 points are possible on this exam

1.) (80 points total) An economist is interested in estimating the price elasticity of petroleum supply in the United States. She reasons that the log of demand (q_t^d) depends on the log of the relative price of petroleum (p_t) and the log of U.S. real GDP (y_t),whereas supply (q_t^s) depends on price and the log of proved petroleum reserves (r_t):

$$\begin{aligned} q_t^d &= ap_t + by_t + u_t \\ q_t^s &= cp_t + dr_t + v_t. \end{aligned}$$

We assume that the market is in equilibrium so that $q_t^d = q_t^s = q_t$. Suppose that the vector $(q_t, p_t, y_t, r_t)'$ is stationary and ergodic and that the vector $(u_t, v_t)'$ is a martingale difference sequence with respect to $u_{t-1}, u_{t-2}, ..., u_1, v_{t-1}, v_{t-2}, ..., v_1, r_t, r_{t-1}, ..., r_1, y_t, y_{t-1}, ..., y_1$ with $E(u_t^2) = \sigma_u^2$, $E(v_t^2) = \sigma_v^2$, $E(u_t v_t = 0)$, and

$$E\left\{ \begin{bmatrix} u_t \\ v_t \end{bmatrix} \middle| y_t, r_t \right\} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

a.) (30 points) Write down a formula for a consistent estimate of c. Describe the steps you could follow in order to generate this estimate from an OLS computer package.

b.) (30 points) Derive the asymptotic distribution of this estimate of c. (Hint: it's ok to check your results against the information given in question (3) below, if helpful for you). To derive this expression, did you need to make any additional assumptions in addition to those specified above?

c.) (20 points) Describe the statistic that you would calculate in order to conduct a test of the hypothesis $c = 0$. For what value of this statistic would you reject this hypothesis against the two-sided alternative $c \neq 0$?

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2.) (90 points total) Consider the regression model $y_t = \beta_1 + \beta_2 x_t + u_t$ where $E(u_t^2|x_t) = \exp(\alpha_1 + \alpha_2 x_t)$ and $E[\log(u_t^2)|x_t] = \alpha_3 + \alpha_1 x_t$.

a.) (30 points) Write the expressions for the OLS estimates of β_1 and β_2. You may write these in vector or matrix form as long as you are have written out explicitly what the terms are in any vectors or matrices that you use.

b.) (15 points) Write the expression for the White or heteroskedasticity-robust standard errors for the estimates you proposed in part (a).

c.) (20 points) Assuming that the values of the parameters $(\beta_1, \beta_2, \alpha_1, \alpha_2, \alpha_3)'$ are not known but must be estimated, give the expression for the GLS estimate of β_1 and β_2.

d.) (15 points) Write an expression for an estimate of the standard errors for the values of $\hat{\beta}_1$ and $\hat{\beta}_2$ that you proposed in part (d).

e.) (10 points) Can you compare the plim of the standard error for b_2 that you proposed in (b) with the plim of the standard error for $\hat{\beta}_2$ that you proposed in (d)?

3.) (55 points total) If $E[\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)] = \mathbf{0}$, the GMM estimate of $\boldsymbol{\theta}$ is defined as the value that minimizes

$$T[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]'\hat{\mathbf{S}}^{-1}[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]$$

where

$$\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1} \sum_{t=1}^{T} \mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

and $\hat{\mathbf{S}}$ is a consistent estimate of

$$\mathbf{S} = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} \sum_{v=-\infty}^{\infty} E\left\{[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)]\left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v})\right]'\right\}.$$

The usual asymptotic distribution of the GMM estimator is

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

where $\mathbf{V}$ can be estimated consistently from

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}}\hat{\mathbf{S}}^{-1}\hat{\mathbf{D}}')^{-1}$$

$$\hat{\mathbf{D}}' = \left.\frac{\partial \mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}.$$

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Consider estimation of a vector system of the form

$$E\left\{\beta\left[(C_{t+1}/C_t)^{\gamma}\right](1+r_t)\mathbf{x}_t\right\} = E(\mathbf{x}_t)$$

where C_{t+1}/C_t is the growth rate of consumption, r_t is the rate of return between t and $t+1$, and $\mathbf{x}_t = (1, (C_t/C_{t-1}), (C_{t-1}/C_{t-2}), r_{t-1}, r_{t-2})'$, β is a parameter relating to a consumer's rate of time preference, and γ is a parameter related to a consumer's level of risk aversion.

a.) (5 points) Show how the above first-order condition could be motivated from the Euler equation for a dynamic optimization problem of the form

$$U'(C_t) = E_t[\beta U'(C_{t+1})(1+r_t)].$$

b.) (30 points) Give explicit expressions for the values of $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$ and $\hat{\mathbf{D}}'$ for this particular example and propose an estimate of $\hat{\mathbf{S}}$.

c.) (10 points) Write down the value of the statistic that you would use to test the hypothesis joint hypothesis that $\gamma = 0.5$ and $\beta = 0.98$. What critical value would you use to test the hypothesis?

c.) (10 points) Write down a statistic you could use to test the overall validity of the specification, and indicate what critical value you would use to test the hypothesis.

4.) (25 points) State (but do not prove) the Gauss-Markov theorem. Recall that the mathematical statement of a theorem must include a statement of the specific mathematical conditions under which the theorem holds (so don't say something like "assuming the classical regression model.")