Econon 220B

Final Exam
Winter 2006

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 250 points are possible on this exam

1.) (25 points) State (but do not prove) the Gauss-Markov Theorem.

2.) (80 points total) Consider the following model:

$$\underset{(T\times 1)}{\mathbf{y}} = \underset{(T\times k)}{\mathbf{X}}\underset{(k\times 1)}{\boldsymbol{\beta}} + \underset{(T\times 1)}{\mathbf{u}}$$

$$E(\mathbf{u}|\mathbf{X}) = \mathbf{0}$$

$$E(\mathbf{u}\mathbf{u}'|\mathbf{X}) = \mathbf{V}(\boldsymbol{\alpha}, \mathbf{X})$$

$$\text{plim } T^{-1}(\mathbf{X}'\mathbf{X}) = \mathbf{Q} \text{ where } \mathbf{Q} \text{ has rank } k$$

$$\text{plim } T^{-1}(\mathbf{X}'\mathbf{u}) = \mathbf{0}.$$

You need not prove or derive any of the following statements; just give the expression and answer "yes" or "no".

a.) (30 points) Give the formula for the OLS estimate $\mathbf{b}$. Is $\mathbf{b}$ unbiased? Is $\mathbf{b}$ consistent?

b.) (30 points) Suppose that the value of $\boldsymbol{\alpha}$ is known. Give the formula for the GLS estimate $\hat{\boldsymbol{\beta}}_{GLS}$. Is $\hat{\boldsymbol{\beta}}_{GLS}$ biased? Is $\hat{\boldsymbol{\beta}}_{GLS}$ consistent?

c.) (20 points) If instead $\boldsymbol{\alpha}$ has to be estimated consistently from the data (so that $\hat{\boldsymbol{\alpha}}_T \xrightarrow{p} \boldsymbol{\alpha}$), how would that change your answers to part (b)?

3.) (90 points total) Consider the following model:

$$y_t = \mathbf{z}_t^{'}\boldsymbol{\beta} + u_t$$

$$E(\mathbf{x}_t\mathbf{z}_t^{'}) = \mathbf{A}$$

$$E(\mathbf{x}_t\mathbf{x}_t^{'}) = \mathbf{B}$$

$$E(u_t^2) = \sigma^2$$

$$E(u_t^2\mathbf{x}_t\mathbf{x}_t^{'}) = \sigma^2\mathbf{B}$$

Here $\mathbf{z}_t$, $\boldsymbol{\beta}$, and $\mathbf{x}_t$ are each $(k \times 1)$ vectors, $\mathbf{A}$ and $\mathbf{B}$ each are $(k \times k)$ matrices of rank k, the variables $\{\mathbf{x}_t, \mathbf{z}_t, y_t\}$ are stationary and ergodic and $\mathbf{x}_t u_t$ is a martingale difference sequence. The instrumental variables estimator is defined by

$$\hat{\boldsymbol{\beta}}_{IV} = \left(\sum_{t=1}^{T} \mathbf{x}_t\mathbf{z}_t^{'}\right)^{-1} \left(\sum_{t=1}^{T} \mathbf{x}_t y_t\right).$$

a.) (30 points) Show that $\hat{\boldsymbol{\beta}}_{IV}$ is consistent.

b.) (30 points) Show that $\sqrt{T}(\hat{\boldsymbol{\beta}}_{IV} - \boldsymbol{\beta}) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$. Find the value of $\mathbf{V}$.

c.) (30 points) Describe how you could use result (b) to construct a test of $H_0 : \mathbf{R}\boldsymbol{\beta} = \mathbf{r}$. If possible, give the expression you would use for the actual test statistic and describe the conditions under which you would reject H_0.

4.) (55 points total) If $E[\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)] = \mathbf{0}$, the GMM estimate of $\boldsymbol{\theta}$ is defined as the value that minimizes

$$T[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]'\hat{\mathbf{S}}^{-1}[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]$$

where

$$\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1}\sum_{t=1}^{T}\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

and $\hat{\mathbf{S}}$ is a consistent estimate of

$$\mathbf{S} = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\sum_{v=-\infty}^{\infty} E\left\{[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)]\left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v})\right]'\right\}.$$

The usual asymptotic distribution of the GMM estimator is

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

where $\mathbf{V}$ can be estimated consistently from

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}}\hat{\mathbf{S}}^{-1}\hat{\mathbf{D}}')^{-1}$$

$$\hat{\mathbf{D}}' = \left.\frac{\partial \mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}.$$

Consider the following model:

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + u_t$$

$$E(u_t\mathbf{x}_t) = \mathbf{0}$$

$$E(\mathbf{x}_t\mathbf{x}_t') = \mathbf{B}$$

$$E(u_t^2\mathbf{x}_t\mathbf{x}_t') = \mathbf{W}.$$

Do not assume that $\mathbf{W} = E(u_t^2)E(\mathbf{x}_t\mathbf{x}_t')$.

a.) (20 points) Use the above expressions to find the GMM estimator $\hat{\boldsymbol{\beta}}_{GMM}$ for this model.

b.) (20 points) Use the above expressions to find the GMM estimator for the variance of $\hat{\boldsymbol{\beta}}_{GMM}$.

c.) (15 points) Use result (b) to give a formula that might be used to construct a standard error for the first element of the vector $\hat{\boldsymbol{\beta}}_{GMM}$.