Econ 220B

Final Exam
Winter 2007

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 250 points are possible on this exam

1.) (115 points total) Consider the following regression model,

$$y_t = \alpha + \boldsymbol{\gamma}'\mathbf{z}_t + \varepsilon_t$$

for $\mathbf{z}_t$ a $(q \times 1)$ vector of explanatory variables. Let $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_T)'$ and suppose that

$$\boldsymbol{\varepsilon}|\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_T \sim N(\mathbf{0}, \sigma^2\mathbf{I}_T).$$

a.) (20 points) Write down the formula that characterizes the OLS estimate of $\boldsymbol{\gamma}$ as a function of $\{y_1, ..., y_T, \mathbf{z}_1, ..., \mathbf{z}_T\}$.

b.) (20 points) The OLS F test of the null hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$ can be written as

$$F = \frac{(SSR_R - SSR_U)/m}{SSR_U/(T-k)}$$

where SSR_R denotes the restricted sum of squared residuals and SSR_U denotes the unrestricted sum of squared residuals. What are the values of m and k in this case? Explain exactly which regressions you would use to calculate SSR_R and SSR_U.

c.) (20 points) What additional assumptions (if any) beyond those stated above would you need in order to conclude that F has an exact $F(n_1, n_2)$ distribution? What are the values of n_1 and n_2?

d.) (20 points) The centered R^2 for a regression is defined by

$$R^2 = 1 - \frac{\sum_{t=1}^T e_t^2}{\sum_{t=1}^T (y_t - \overline{y})^2}$$

for $\overline{y}$ the sample mean of $y_1, ..., y_T$ and e_t the OLS residual for observation t. Calculate R^2 as a function of the F statistic in part (b).

e.) (35 points) Use the results of (d) to derive the asymptotic distribution of R^2 under the null hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$. What additional assumptions (if any) beyond those you stated in (c) did you need to prove this result?

2.) (50 points total) Consider the following regression model. A variable y_t is hypothesized to depend on the true value of a zero-mean scalar x_t^* according to

$$y_t = \beta x_t^* + \varepsilon_t.$$

Let $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, .., \varepsilon_T)'$ and $\mathbf{x}^* = (x_1^*, x_2^*, ..., x_T^*)'$. We assume that $\boldsymbol{\varepsilon}|\mathbf{x}^* \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_T)$. The variable x_t^* is not observed directly by the econometrician, who instead has data on a proxy x_t characterized by

$$x_t = x_t^* + v_t$$

where the measurement error v_t is Gaussian and is independent of both x_t^* and ε_t:

$$\mathbf{v}|\boldsymbol{\varepsilon}, \mathbf{x}^* \sim N(\mathbf{0}, \tau^2 \mathbf{I}_T).$$

The vector $(x_t^*, v_t, \varepsilon_t)'$ is stationary and ergodic with variance-covariance matrix given by

$$E \begin{bmatrix} x_t^* \\ v_t \\ \varepsilon_t \end{bmatrix} \begin{bmatrix} x_t^* & v_t & \varepsilon_t \end{bmatrix} = \begin{bmatrix} q^2 & 0 & 0 \\ 0 & \tau^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

a.) (15 points) Does this example satisfy the assumptions of the classical regression model? Explain.

b.) (35 points) Calculate the plim of the OLS estimate b where b is given by

$$b = \left(\sum_{t=1}^{T} x_t^2 \right)^{-1} \left(\sum_{t=1}^{T} x_t y_t \right)$$

3.) (85 points total) In answering this question, you may find some of the following results for GMM estimation useful:

$$E[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)] = \mathbf{0}$$

$$\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T) = T^{-1} \sum_{t=1}^{T} \mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t)$$

$$\hat{\boldsymbol{\theta}}_{GMM} = \arg \min_{\boldsymbol{\theta}} T[\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]' \mathbf{S}^{-1} [\mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)]$$

$$\mathbf{S} = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} \sum_{v=-\infty}^{\infty} E\left\{ [\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_t)] \left[\mathbf{h}(\boldsymbol{\theta}_0, \mathbf{w}_{t-v}) \right]' \right\}.$$

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_{GMM} - \boldsymbol{\theta}_0) \xrightarrow{L} N(\mathbf{0}, \mathbf{V})$$

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}} \hat{\mathbf{S}}^{-1} \hat{\mathbf{D}}')^{-1}$$

$$\hat{\mathbf{D}}' = \left. \frac{\partial \mathbf{g}(\boldsymbol{\theta}; \mathbf{Y}_T)}{\partial \boldsymbol{\theta}'} \right|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}}.$$

In this problem you are invited (but not required) to use these GMM results to consider the following model,

$$y_t = \mathbf{z}_t' \boldsymbol{\beta} + u_t$$

where $\mathbf{z}_t$ and $\mathbf{x}_t$ are both $(k \times 1)$ vectors and $(\mathbf{x}_t', \mathbf{z}_t', u_t)'$ is stationary and ergodic. In addition, you can also assume the following:

$$T^{-1/2} \sum_{t=1}^{T} \mathbf{x}_t u_t \xrightarrow{L} N(\mathbf{0}, \mathbf{S}) \quad \text{(A)}$$

$$E(\mathbf{z}_t u_t) \neq \mathbf{0} \quad \text{(B)}$$

$$E(\mathbf{x}_t u_t) = \mathbf{0} \quad \text{(C)}$$

$$E(\mathbf{x}_t \mathbf{z}_t') = \mathbf{H} \text{ has rank } k \quad \text{(D)}$$

$$E(\mathbf{x}_t u_t \mathbf{x}_{t-v}' u_{t-v}) = \rho^v \boldsymbol{\Gamma}_0 \quad \text{(E)}$$

where for purposes of this question you can treat $\boldsymbol{\Gamma}_0$ as a known positive definite symmetric matrix and ρ as a known scalar satisfying $0 < \rho < 1$.

a.) (5 points) Condition (A) is something that comes up sufficiently often in econometrics that we usually refer to it by a particular name. What is that name?

b.) (5 points) Condition (B) is something that comes up sufficiently often in econometrics that we usually refer to it by a particular name. What is that name?

c.) (5 points) Condition (C) is something that comes up sufficiently often in econometrics that we usually refer to it by a particular name. What is that name?

d.) (5 points) Condition (D) is something that comes up sufficiently often in econometrics that we usually refer to it by a particular name. What is that name?

e.) (5 points) Condition (E) is something that comes up sufficiently often in econometrics that we usually refer to it by a particular name. What is that name?

f.) (20 points) Calculate the GMM estimator of $\boldsymbol{\beta}$ (or, if you're not able to do that, suggest another estimator of $\boldsymbol{\beta}$ that you think might be a good one to use).

g.) (20 points) Use the general results on GMM estimation to calculate the asymptotic variance of your estimator $\hat{\boldsymbol{\beta}}_{GMM}$ (or, if you're not able to do that, derive the asymptotic distribution of the estimator you proposed in part (f)).

h.) (20 points) Write down the statistic that you could use to test the null hypothesis that β_1 (the first element of the vector $\boldsymbol{\beta}$) is equal to 3. What would you expect the asymptotic distribution of this statistic to be? (You don't have to prove that this is the distribution).