

Econ 220B

Answers to 220B Winter 2007 Final Exam

1a.)

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} T & \Sigma \mathbf{z}'_t \\ \Sigma \mathbf{z}_t & \Sigma \mathbf{z}_t \mathbf{z}'_t \end{bmatrix}^{-1} \begin{bmatrix} \Sigma y_t \\ \Sigma \mathbf{z}_t y_t \end{bmatrix}$$

or

$$\hat{\gamma} = \left[ \Sigma (\mathbf{z}_t - \bar{\mathbf{z}})(\mathbf{z}_t - \bar{\mathbf{z}})' \right]^{-1} \left[ \Sigma (\mathbf{z}_t - \bar{\mathbf{z}})(y_t - \bar{y}) \right]$$

1b.)  $m = q$

$$k = q + 1$$

$SSR_R$  from regression of  $y_t$  on 1

$SSR_U$  from regression described in 1a

1c.) no further assumptions needed

$$n_1 = q$$

$$n_2 = T - q - 1$$

1d.)  $R^2 = 1 - SSR_U/SSR_R$

$$m(T - k)^{-1}F = -1 + SSR_R/SSR_U$$

$$\frac{1}{1 + m(T - k)^{-1}F} = \frac{SSR_U}{SSR_R}$$

$$R^2 = 1 - \frac{1}{1 + m(T - k)^{-1}F} = \frac{m(T - k)^{-1}F}{1 + m(T - k)^{-1}F}$$

1e.)

$$R^2 = \frac{mF}{(T - k) + mF} \xrightarrow{p} 0$$

$$(T - k)R^2 = \frac{(T - k)mF}{(T - k) + mF} = \frac{mF}{1 + mF/(T - k)} \xrightarrow{p} mF \xrightarrow{L} \chi^2(m)$$

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2a.) Yes if we regressed  $y_t$  on  $x_t^*$ , no if we regressed  $y_t$  on  $x_t$

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{u}$$

$$\mathbf{u} = \boldsymbol{\varepsilon} - \mathbf{v}\beta$$

$$E(\mathbf{u}|\mathbf{x}) \neq \mathbf{0}$$

$$2b.) [T^{-1}\Sigma(x_t^* + v_t)^2]^{-1} [T^{-1}\Sigma(x_t^* + v_t)(\beta x_t^* + \varepsilon_t)]$$

$$\xrightarrow{p} (q^2 + \tau^2)^{-1}(q^2\beta) = bq^2/(q^2 + \tau^2)$$

3a.) CLT

b.) endogenous regressors

c.) valid instrument

d.) relevant instrument

e.) serial correlation

f.)  $\Sigma \mathbf{x}_t(y_t - \mathbf{z}_t'\boldsymbol{\beta}) = \mathbf{0}$

$$\hat{\boldsymbol{\beta}} = (\Sigma \mathbf{x}_t \mathbf{z}_t')^{-1} (\Sigma \mathbf{x}_t y_t)$$

g.)  $\mathbf{h} = \mathbf{x}_t(y_t - \mathbf{z}_t'\boldsymbol{\beta})$

$$\hat{\mathbf{D}}' = -T^{-1}\Sigma \mathbf{x}_t \mathbf{z}_t'$$

$$\mathbf{S} = \left[ \frac{1}{1-\rho} + \frac{\rho}{1-\rho} \right] \boldsymbol{\Gamma}_0 = \left[ \frac{1+\rho}{1-\rho} \right] \boldsymbol{\Gamma}_0$$

$$\hat{\mathbf{V}} = \left[ \left( T^{-1}\Sigma \mathbf{z}_t \mathbf{x}_t' \right) \left( \frac{1+\rho}{1-\rho} \right) \boldsymbol{\Gamma}_0^{-1} \left( T^{-1}\Sigma \mathbf{x}_t \mathbf{z}_t' \right) \right]$$

h.)

$$\frac{(\hat{\beta}_1 - 3)}{\sqrt{\mathbf{e}_1'(\hat{\mathbf{V}}/T)\mathbf{e}_1}} \approx N(0, 1)$$