Econon 220B

Answers to 220B Winter 2008 Final Exam

1a.) $(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$

b.) $(\mathbf{Rb})'\left[\mathbf{R}s^2(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{R}'\right]^{-1}(\mathbf{Rb})$

$\mathbf{R} = \begin{bmatrix} 1 & -2 \end{bmatrix}$

$F(1, T-2)$

c.)

$$\frac{\mathbf{Rb}}{\sqrt{\mathbf{R}s^2(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'}} \sim t(T-k)$$

d.) Replace $s^2(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}$ with

$$(\mathbf{X}'\mathbf{X})^{-1}\sum_{t=1}^{T} e_t^2 \mathbf{x}_t \mathbf{x}_t'(\mathbf{X}'\mathbf{X})^{-1}$$

e.) Let $g(\mathbf{b}) = \log b_1 - 2\log b_2$

$\mathbf{g}' = \begin{bmatrix} \frac{1}{b_1} & \frac{-2}{b_2} \end{bmatrix}$

$g(\mathbf{b})^2 / \left[\mathbf{g}' s^{\mathbf{2}}(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{g}\right] \approx \chi^1(1)$

2a.) Factors such as education might be correlated with both ε_i and s_i

b.) $E\left(\mathbf{x}_i \begin{bmatrix} 1 & s_i \end{bmatrix}\right)$ has rank 2. To test, regress s_i on $\mathbf{x}_i$ and test whether coefficients on $\mathbf{x}_i$ are all the same

c.) $E(\mathbf{x}_i \varepsilon_i) = \mathbf{0}$, could test with GMM test of overidentifying restrictions

d.) Step 1: regress s_i on $\mathbf{x}_i$, save fitted values $\hat{s}_i$

Step 2: regress y_i on constant and $\hat{s}_i$

3a.)

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{N}{p} - \frac{T-N}{1-p} = 0 \text{ or } \hat{p} = N/T$$

b.) $\sqrt{T}(\hat{p} - p) \xrightarrow{L} N(0, p(1-p))$

c.)

$$h_t = \frac{y_t}{p} - \frac{1-y_t}{1-p} = \frac{y_t - p}{p(1-p)}$$

d.)

$$E(h_t) = \frac{E(y_t) - p}{p(1-p)} = \frac{p-p}{p(1-p)} = 0$$

e.)

$$S = E(h_t^2) = E\left[\frac{y_t^2 - 2py_t + p^2}{p^2(1-p)^2}\right] = \left[\frac{p - 2p^2 + p^2}{p^2(1-p)^2}\right] = \frac{1}{p(1-p)}$$

f.)

$$\frac{\partial h_t}{\partial p} = \frac{-y_t}{p^2} - \frac{(1-y_t)}{(1-p)^2}$$

$$\frac{\partial g}{\partial p} = -T^{-1}\sum_{t=1}^{T}\left[\frac{y_t}{p^2} + \frac{(1-y_t)}{(1-p)^2}\right]$$

$$\left.\frac{\partial g}{\partial p}\right|_{p=\hat{p}} = -\left[\frac{1}{\hat{p}} + \frac{1}{1-\hat{p}}\right] = -\frac{1}{\hat{p}(1-\hat{p})}$$

g.)

$$\hat{V} = \left\{[\hat{p}(1-\hat{p})]^{-1}[\hat{p}(1-\hat{p})][\hat{p}(1-\hat{p})]^{-1}\right\}^{-1} = \hat{p}(1-\hat{p})$$

which is same answer as for (b)

h.) Yes because the moment condition $E(h_t) = 0$ would still hold

i.) For b, plim is same but distribution is now $\sqrt{T}(\hat{p} - p) \xrightarrow{L} N(0, Q)$ for

$$Q = \sum_{j=-\infty}^{\infty} E(y_t - p)(y_{t-j} - p)$$

For (e) we have

$$\begin{aligned} S &= \sum_{j=-\infty}^{\infty} E\left[\frac{y_t - p}{p(1-p)}\right]\left[\frac{y_{t-j} - p}{p(1-p)}\right] \\ &= \left[\frac{1}{p^2(1-p)^2}\right] Q \end{aligned}$$

and

$$V = \left\{[p(1-p)]^{-1}\left[\frac{Q}{p^2(1-p)^2}\right]^{-1}[p(1-p)]^{-1}\right\}^{-1} = Q$$

again the same answer.