Econ 220B, Winter 2007
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Answers to Practice Midterm Exam

1a.) No, because $\sum e_t^2$ cannot go up when you add one more variable and $\sum y_t^2$ does not change.

b.) No, because $\sum e_t^2$ cannot go up when you add one more variable and $\sum (y_t - \overline{y})^2$ does not change.

2a.) $E(\varepsilon_t \mathbf{x}_t | \mathbf{x}_t, \mathbf{x}_{t-1}, ..., \mathbf{x}_1, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_1) = \mathbf{0}$ and by Law of Iterated Expectations, $E(\varepsilon_t \mathbf{x}_t | \mathbf{x}_{t-1}, ..., \mathbf{x}_1, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_1) = \mathbf{0}$

b.) Not necessarily since no guarantee that $E(\boldsymbol{\varepsilon}|\mathbf{X}) = \mathbf{0}$

c.) $F = (b_1 + b_2)^2 / \left[s^2 \mathbf{q}' \left(\sum \mathbf{x}_t \mathbf{x}_t' \right)^{-1} \mathbf{q} \right]$ for $\mathbf{q} = (1, 1, 0, ..., 0)'$, $\mathbf{b} = \left(\sum \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(\sum \mathbf{x}_t y_t\right)$ and $s^2 = (T-k)^{-1} \sum (y_t - \mathbf{x}_t' \mathbf{b})^2$

d.)

$$F = \frac{T(b_1 + b_2)^2 / \left[\sigma^2 \mathbf{q}' \left(T^{-1} \sum \mathbf{x}_t \mathbf{x}_t' \right)^{-1} \mathbf{q} \right]}{s^2 / \sigma^2}$$

Numerator: $\sqrt{T}(\mathbf{b}_T - \boldsymbol{\beta}) = \left[T^{-1} \sum \mathbf{x}_t \mathbf{x}_t' \right]^{-1} \left[T^{-1/2} \sum \mathbf{x}_t \varepsilon_t \right] \xrightarrow{L} \mathbf{Q}^{-1} \mathbf{z} \sim N(\mathbf{0}, \sigma^2 \mathbf{Q}^{-1})$ because $\mathbf{z} \sim N(\mathbf{0}, \sigma^2 \mathbf{Q})$, from which $T(\mathbf{q}' \mathbf{b}_T)^2 / [\sigma^2 \mathbf{q}' \mathbf{Q}^{-1} \mathbf{q}] \xrightarrow{L} \chi^2(1)$. Hence numerator is asymptotically $\chi^2(1)$. Denominator: $s_T^2 \xrightarrow{p} \sigma^2$ as shown in class, so plim of denominator is unity. Hence $F_T \xrightarrow{L} \chi^2(1)$.

3a.) GLS estimator has smaller variance than b.

b.) Want to simplify $\hat{\boldsymbol{\beta}}_{GLS} = [\mathbf{X}' \mathbf{V}(\mathbf{X})^{-1} \mathbf{X}]^{-1} [\mathbf{X}' \mathbf{V}(\mathbf{X})^{-1} \mathbf{y}]$. Here

$$\mathbf{V}(\mathbf{X})^{-1} = \begin{bmatrix} 1/x_1 & 0 & 0 & \cdots & 0 \\ 0 & 1/x_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1/x_T \end{bmatrix}$$

$$\mathbf{X}' \mathbf{V}(\mathbf{X})^{-1} \mathbf{X} = \sum x_t$$

$$\mathbf{X}' \mathbf{V}(\mathbf{X})^{-1} \mathbf{y} = \sum y_t$$

$$\hat{\beta}_{GLS} = \frac{\sum y_t}{\sum x_t}.$$

c.) $\tilde{\mathbf{y}} = \mathbf{P}^{-1} \mathbf{y}$ where $\mathbf{P}$ is diagonal with $x_t^{1/2}$ in row t, col t position, meaning $\tilde{y}_t = y_t / x_t^{1/2}$ and $\tilde{x}_t = x_t / x_t^{1/2} = x_t^{1/2}$. Hence do OLS regression of $\tilde{y}_t$ on $\tilde{x}_t$.