Econ 220B, Winter 2007
James Hamilton

Practice Midterm Exam

DIRECTIONS: No books or notes of any kind are allowed. Answer all questions on separate paper. 150 points are possible on this exam.

1a.) (5 points) Can the uncentered R^2 ever go down if you add one more variable to the regression? If so, how? If not, why not?

b.) (5 points) Can the centered R^2 ever go down if you add one more variable to the regression? If so, how? If not, why not?

2.) Suppose $y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t$ where $\mathbf{x}_t$ is a $(k \times 1)$ vector of explanatory variables and

$$\varepsilon_t | \mathbf{x}_t, \mathbf{x}_{t-1}, ..., \mathbf{x}_1, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_1 \sim N(0, \sigma^2)$$

with $\{\varepsilon_t, \mathbf{x}_t\}$ stationary and ergodic and $E(\varepsilon_t^2 \mathbf{x}_t \mathbf{x}_t') = \sigma^2 \mathbf{Q}$. You can further assume that $T^{-1} \sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t' \xrightarrow{p} \mathbf{Q}$ which has rank k. The OLS estimates are given by $\mathbf{b} = \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(\sum_{t=1}^T \mathbf{x}_t y_t\right)$ and $s^2 = (T-k)^{-1} \sum_{t=1}^T (y_t - \mathbf{x}_t'\mathbf{b})$.

a.) (10 points) Prove that under the above assumptions, $\{\varepsilon_t \mathbf{x}_t\}_{t=1}^T$ is a martingale difference sequence.

b.) (10 points) Under these assumptions, is $\mathbf{b}$ an unbiased estimate of $\boldsymbol{\beta}$?

c.) (20 points) Suppose you wanted to test the hypothesis that $\beta_1 + \beta_2 = 0$. Write down the formula for the F test of this null hypothesis. Note that everything on the right-hand side of your formula should be an explicit function of $\{y_t, \mathbf{x}_t\}_{t=1}^T$.

d.) (40 points) Calculate the asymptotic distribution of the statistic you proposed in step (c). Show all the steps in your derivation. Hint: if you could not answer step (c), you can still get partial credit for step (d) by deriving the asymptotic distribution of a variable with an F distribution.

3.) Suppose $y_t = x_t\beta + \varepsilon_t$ for x_t a single positive explanatory variable, or, in vector form, $\mathbf{y} = \mathbf{x}\beta + \varepsilon$ for $\mathbf{y}, \mathbf{x}$ and $\boldsymbol{\varepsilon}$ each $(T \times 1)$ vectors and β a scalar. Suppose that $E(\varepsilon|\mathbf{x}) = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'|\mathbf{x}) = \sigma^2 \mathbf{V}(\mathbf{x})$ where the $(T \times T)$ diagonal matrix $\mathbf{V}(\mathbf{x})$ is given by

$$\mathbf{V}(\mathbf{x}) = \begin{bmatrix} x_1 & 0 & 0 & \cdots & 0 \\ 0 & x_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & x_T \end{bmatrix}.$$

a.) (10 points) The OLS estimate of β is given by the expression

$$b = \frac{\sum_{t=1}^T x_t y_t}{\sum_{t=1}^T x_t^2}.$$

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What would be the major advantages of using the GLS estimate of β instead of b?

b.) (20 points) Find an expression for the GLS estimate of β in the same form as the OLS formula written above. Note that you need the actual formula for this particular case, meaning that the notation $\mathbf{V}(\mathbf{x})$ or some matrix $\mathbf{P}$ should not appear in your answer.

c.) (10 points) Describe an OLS regression of a transformed $\tilde{y}_t$ on a transformed $\tilde{x}_t$ that would be one way to arrive at the GLS estimate you found in part (b). Again, give details relevant for actually implementing the procedure in this particular case.