VAR's: Estimation, forecasting and hypothesis tests

A. Principles of forecasting

Suppose we want to forecast y_{1t} based on:

$y_{1,t-1}, y_{1,t-2}, \ldots, y_{1,t-p}$

$y_{2,t-1}, y_{2,t-2}, \ldots, y_{2,t-p}$

$\vdots$

$y_{n,t-1}, y_{n,t-2}, \ldots, y_{n,t-p}$

deterministic functions of t

$(1, t, \cos(\pi t/6),$ seasonal dummies$)$

Let $\underset{(n\times 1)}{\mathbf{y}_t} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$

$\underset{(k\times 1)}{\mathbf{x}_t} = (1, \mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \ldots, \mathbf{y}'_{t-p})'$

$k = np + 1$

Suppose we consider linear forecast

$\hat{y}_{1t|t-1} = \boldsymbol{\beta}'\mathbf{x}_t$

Best forecast: value of $\boldsymbol{\beta}$ that minimizes

$E(y_{1t} - \boldsymbol{\beta}'\mathbf{x}_t)^2$

Proposition: If $\mathbf{y}_t$ is covariance-stationary and $E(\mathbf{x}_t\mathbf{x}'_t)$ is nonsingular, then optimal forecast uses

$\boldsymbol{\beta}^* = E(\mathbf{x}_t\mathbf{x}'_t)^{-1}E(\mathbf{x}_t y_t)$

Definition: The optimal linear forecast,

$\hat{y}_{1t|t-1} = \boldsymbol{\beta}^{*\prime}\,\mathbf{x}_t,$

is called the "population linear projection" of y_{1t} on $\mathbf{x}_t$

Definition: Ordinary least squares (OLS) estimate is given by

$$\hat{\boldsymbol{\beta}} = \left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(\sum_{t=1}^{T} \mathbf{x}_t y_t\right)$$

Proposition: If $\mathbf{y}_t$ is ergodic, then

$$\hat{\boldsymbol{\beta}} \xrightarrow{p} \boldsymbol{\beta}^*$$

Proof: (Law of Large Numbers)

$$\hat{\boldsymbol{\beta}} = \left(T^{-1} \sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(T^{-1} \sum_{t=1}^{T} \mathbf{x}_t y_t\right)$$

$$\xrightarrow{p} E(\mathbf{x}_t \mathbf{x}_t')^{-1} E(\mathbf{x}_t y_t)$$

Example 1: Single equation estimation

y_{1t} = real GDP growth

y_{2t} = inflation

y_{3t} = fed funds rate

y_{4t} = rate of growth of M2

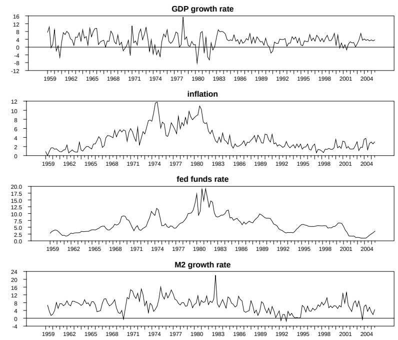
Example 1: Single equation estimation

- cal 1947 1 4
- all 0 2006:4
- open data gdp_data.prn
- data(org=obs) 1947:1 2005:3 datex gdplev pricelev
- open data fed_data.prn
- data(org=obs) 1959:1 2005:3 date2 fedfunds M2

1947	1570.5	15.105
1947.25	1568.7	15.329
1947.5	1568	15.597
1947.75	1590.9	15.989

- set gdpch = 400*log(gdplev(t)/gdplev(t-1))
- set inflation = 400*log(pricelev(t)/pricelev(t-1))
- set mgrow = 400*log(M2(t)/M2(t-1))
- smpl 1959:1 2005:3

- spgraph(vfields=4)
- graph(header='GDP growth rate',dates) 1
- # gdpch
- graph(samesize,header='inflation',dates) 1
- # inflation
- graph(samesize,header='fed funds rate',dates) 1
- # fedfunds
- graph(samesize,header='M2 growth rate',dates) 1
- # mgrow
- spgraph(done)

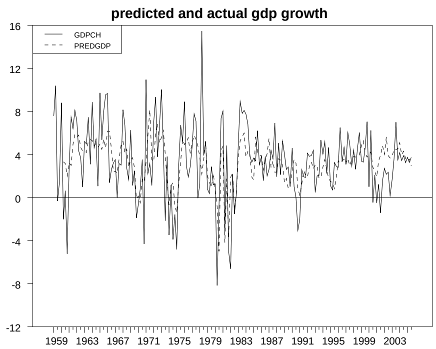


- linreg gdpch
- # constant gdpch{1 to 4} inflation{1 to 4} fedfunds{1 to 4} mgrow{1 to 4}

- Linear Regression - Estimation by Least Squares
- Dependent Variable GDPCH
- Quarterly Data From 1959:01 To 2005:03
- Usable Observations 182 Degrees of Freedom 165
- Total Observations 187 Skipped/Missing 5
- Centered R**2 0.325262 R Bar **2 0.259833
- Uncentered R**2 0.650477 T x R**2 118.387
- Mean of Dependent Variable 3.2792963257
- Std Error of Dependent Variable 3.4090248280
- Standard Error of Estimate 2.9328850575
- Sum of Squared Residuals 1419.2994355
- Regression F(16,165) 4.9712
- Significance Level of F 0.00000003
- Log Likelihood -445.15280
- Durbin-Watson Statistic 1.917563

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	2.082233813	0.811822124	2.56489	0.01121177
2. GDPCH{1}	0.118067740	0.079053976	1.49351	0.13721407
3. GDPCH{2}	0.203099131	0.080889608	2.51082	0.01300744
4. GDPCH{3}	-0.039822886	0.074458465	-0.53483	0.59348496
5. GDPCH{4}	0.027637404	0.069555777	0.39734	0.69162910
6. INFLATION{1}	0.171477187	0.211777736	0.80970	0.41927673
7. INFLATION{2}	-0.006036365	0.233560625	-0.02584	0.97941223
8. INFLATION{3}	-0.313409525	0.239203890	-1.31022	0.19194273
9. INFLATION{4}	0.027459910	0.217167797	0.12645	0.89953317
10. FEDFUNDS{1}	0.141392084	0.206817856	0.68366	0.49515192
11. FEDFUNDS{2}	-1.012059455	0.228992546	-4.41962	0.00001786
12. FEDFUNDS{3}	0.424541703	0.240408102	1.76592	0.07925855
13. FEDFUNDS{4}	0.247841364	0.219214781	1.13059	0.25986970
14. MGROW{1}	0.077751053	0.089889624	0.86496	0.38831579
15. MGROW{2}	0.086579388	0.094657441	0.91466	0.36170479
16. MGROW{3}	0.103284358	0.091678316	1.12660	0.26154944
17. MGROW{4}	0.002498693	0.085254375	0.02931	0.97665382

- prj predgdp 1960:2 2005:4
- graph(header='predicted and actual gdp growth',dates,key=upleft) 2
- # gdpch
- # predgdp



- print(dates) 2005:1 2005:4 gdpch predgdp

ENTRY	GDPCH	PREDGDP
2005:01	3.73397424782	3.426152642086
2005:02	3.25601091600	3.576184537927
2005:03	3.73391574327	2.933814021196
2005:04	NA	2.721240915629

I.

A. Principles of forecasting
B. Hypothesis testing

So far we assumed:

$\mathbf{y}_t$ is stationary and ergodic

$E(\mathbf{x}_t\mathbf{x}_t')$ is nonsingular

and concluded:

OLS gives consistent estimate of optimal forecast weights

Suppose we want to go further and test a hypothesis, e.g.

$H_0 : y_{2,t-1}, y_{2,t-2}, \ldots, y_{2,t-p}$

do not help forecast y_{1t}

Stronger assumptions for hypothesis testing:

$$\varepsilon_t = y_{1t} - \boldsymbol{\beta}^{*'}\mathbf{x}_t$$

$$E(\varepsilon_t|\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_1) = 0$$

$$E(\varepsilon_t^2) = \sigma^2$$

$$E(\varepsilon_t^4) < \infty$$

Then: all usual OLS t or F tests on $\hat{\beta}$ are valid for drawing conclusions about β^*

Example 2: Checking for lag length

H_0 : need $p-1$ lags

H_A : need p lags

H_0 : coeffs on $y_{1,t-p}, y_{2,t-p}, \ldots, y_{n,t-p}$ are all zero

- linreg gdpch
- # constant gdpch{1 to 5} inflation{1 to 5} fedfunds{1 to 5} mgrow{1 to 5}
- exclude
- # gdpch{5} inflation{5} fedfunds{5} mgrow{5}

- Null Hypothesis : The Following Coefficients Are Zero
- GDPCH Lag(s) 5
- INFLATION Lag(s) 5
- FEDFUNDS Lag(s) 5
- MGROW Lag(s) 5
- F(4,160)= 0.81118 with Significance Level 0.51974009

- linreg gdpch
- # constant gdpch{1 to 4} inflation{1 to 4} fedfunds{1 to 4} mgrow{1 to 4}
- exclude
- # gdpch{4} inflation{4} fedfunds{4} mgrow{4}

- Null Hypothesis : The Following Coefficients Are Zero
- GDPCH Lag(s) 4
- INFLATION Lag(s) 4
- FEDFUNDS Lag(s) 4
- MGROW Lag(s) 4
- F(4,164)= 0.39151 with Significance Level 0.81451782

Example 3: Granger-causality

If coeffs on $y_{2,t-1}, y_{2,t-2}, \ldots, y_{2,t-p}$ are all zero, then we say that

"y_2 does not Granger-cause y_1"

- linreg gdpch
- # constant gdpch{1 to 4} inflation{1 to 4} fedfunds{1 to 4} mgrow{1 to 4}
- exclude
- # inflation{1 to 4}

- Null Hypothesis : The Following Coefficients Are Zero
- INFLATION Lag(s) 1 to 4
- F(4,164)= 0.71171 with Significance Level 0.58502790

- exclude
- # mgrow{1 to 4}

- Null Hypothesis : The Following Coefficients Are Zero
- MGROW Lag(s) 1 to 4
- F(4,164)= 1.97125 with Significance Level 0.10126036

- exclude
- # fedfunds{1 to 4}

- Null Hypothesis : The Following Coefficients Are Zero
- FEDFUNDS Lag(s) 1 to 4
- F(4,164)= 6.18029 with Significance Level 0.00011739

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17. MGROW{4}	0.002498693	0.085254375	0.02931	0.97665382

What if one of our variables (say y_{2t}) were something like stock returns?

- linreg gdpch
- # constant gdpch{1 to 4} stockch{1 to 4}
- exclude
- # stockch{1 to 4}

- Null Hypothesis : The Following Coefficients Are Zero
- STOCKCH Lag(s) 1 to 4
- F(4,172)= 6.85095 with Significance Level 0.00003857

- linreg stockch
- # constant gdpch{1 to 4} stockch{1 to 4}
- exclude
- # gdpch{1 to 4}

- Null Hypothesis : The Following Coefficients Are Zero
- GDPCH Lag(s) 1 to 4
- F(4,172)= 0.91726 with Significance Level 0.45522372

GDP $\nRightarrow$ stocks	stocks $\Rightarrow$ GDP
inflation $\nRightarrow$ stocks	stocks $\nRightarrow$ inflation
fed funds $\nRightarrow$ stocks	stocks $\Rightarrow$ fed funds
money growth $\nRightarrow$ stocks	stocks $\nRightarrow$ money growth

Efficient markets theory:
(1) No variable should forecast stock returns (equivalent to no variable should Granger-cause y_2)
(2) Stock returns reflect forecast of future fundamentals (y_2 Granger-causes GDP and fed funds rate)

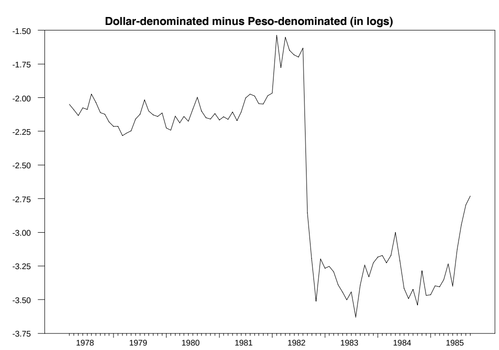
Summary: VAR's can be useful for making statements about forecasting

Interpreting statements about forecasting as statements about causation can be problematic

Variables that can have strong forecasting components:

- stock returns
- interest rates
- exchange rates
- commodity prices

Testing for structural stability



Chow test:

(1) Estimate model over whole sample:

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t \quad \mathbf{x}_t = (k \times 1)$$

save residual sum of squares RSS_0

(2) Estimate model with different coefficients before and after date t_1,

$$y_t = \mathbf{x}_t'\boldsymbol{\beta}_1(\delta_{[t \le t_1]}) + \mathbf{x}_t'\boldsymbol{\beta}_2(\delta_{[t > t_1]}) + \varepsilon_t$$

save RSS_1

(3) Calculate $F(t_1) = \frac{(T-2k)(RSS_0 - RSS_1)}{k\,RSS_1}$

(4) Conclude stable if $F(t_1)$ is below $F(k, T-2k)$ critical value

Example 4: Testing for structural stability

```
• set dum = 1
• set dum 1947:1 1985:4 = 0

• procedure somlags
•     set gdp1 = gdpch(t-1)*dum(t)
•     set gdp2 = gdpch(t-2)*dum(t)
•     set gdp3 = gdpch(t-3)*dum(t)
•     set gdp4 = gdpch(t-4)*dum(t)
•     set inf1 = inflation(t-1)*dum(t)
•     set inf2 = inflation(t-2)*dum(t)
•     set inf3 = inflation(t-3)*dum(t)
•          …
•     set mgrow3 = mgrow(t-3)*dum(t)
•     set mgrow4 = mgrow(t-4)*dum(t)
•  end
```

```
• @somlags
• linreg gdpch
• # constant gdpch{1 to 4}
  inflation{1 to 4} fedfunds{1 to
  4} mgrow{1 to 4} $
•   dum gdp1 gdp2 gdp3 gdp4
  inf1 inf2 inf3 inf4 ff1 ff2 ff3 ff4
  mgrow1 mgrow2 mgrow3
  mgrow4
• exclude
• # dum gdp1 gdp2 gdp3 gdp4
  inf1 inf2 inf3 inf4 ff1 ff2 ff3 ff4
  mgrow1 mgrow2 mgrow3
  mgrow4
```

- Null Hypothesis : The Following Coefficients Are Zero
- DUM
- GDP1
- GDP2
- …
- MGROW3
- MGROW4
- F(17,147)= 0.93285 with Significance Level 0.53706567

Andrews (1993):

(1) Do above calculation for each possible t_1 between first 15% and last 15% of sample.

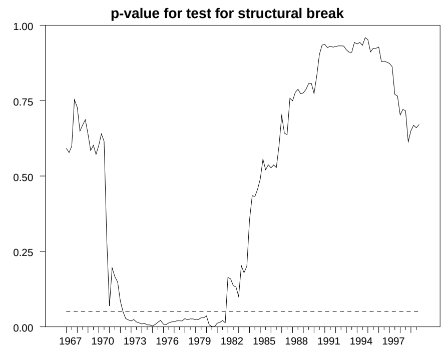
(2) Compare biggest $kF(t_1)$ found with critical value from $\pi_0 = 0.15$ row of Table 1 in Andrews (1993)

```
• do time=1967:1,1999:4
•   set dum = 1
•   set dum 1960:3 time = 0
•   @somlags

•   linreg(noprint) gdpch
•   # constant gdpch{1 to 4} inflation{1 to 4} fedfunds{1 to 4} mgrow{1 to 4} $
•     dum gdp1 gdp2 gdp3 gdp4 inf1 inf2 inf3 inf4 ff1 ff2 ff3 ff4 mgrow1 mgrow2
  mgrow3 mgrow4
•   exclude(noprint)
•   # dum gdp1 gdp2 gdp3 gdp4 inf1 inf2 inf3 inf4 ff1 ff2 ff3 ff4 mgrow1 mgrow2
  mgrow3 mgrow4

•   set qmork time time = %cdstat
•   set qcmork time time = %signif
• end do time
• smpl 1967:1 1999:4
• print(dates) / qmork qcmork
```

- set hq 1967:1 1999:4 = 0.05
- graph(header='p-value for test for structural break') 2
- # qcmork
- # hq



- Maximum value for F-statistic (1980:4) = 2.919
- k = 17
- (2.919)(17) = 49.62

		p = 16			p = 17			p = 18		
π_0	λ	10%	5%	1%	10%	5%	1%	10%	5%	1%
.50	1.00	23.84	26.50	32.00	24.78	27.58	33.45	25.99	28.87	34.81
.49	1.08	25.32	28.17	34.11	26.77	29.72	33.57	27.85	30.78	36.82
.48	1.17	26.22	29.05	35.09	27.65	30.58	36.78	28.76	31.57	37.70
.47	1.27	26.63	29.75	35.67	28.16	31.29	37.38	29.32	32.24	38.65
.45	1.49	27.75	30.99	36.88	29.14	32.18	38.25	30.36	33.17	39.31
.40	2.25	29.34	32.29	38.56	30.82	33.74	39.66	32.11	35.18	40.99
.35	3.45	30.56	33.40	39.18	31.96	34.86	40.01	33.40	36.23	41.84
.30	5.44	31.56	34.41	40.29	32.99	35.83	41.75	34.33	37.09	42.69
.25	9.00	32.33	35.19	41.07	33.80	36.71	42.46	35.27	37.94	43.60
.20	16.00	33.21	35.93	41.79	34.53	37.49	43.27	36.07	38.77	44.09
.15	32.11	33.90	36.66	42.48	35.39	38.12	43.95	36.85	39.55	44.84
.10	81.00	34.72	37.48	43.18	36.19	39.05	44.52	37.64	40.56	45.09
.05	361.00	35.81	38.81	44.28	37.24	40.05	45.39	38.68	41.36	46.49

49.62 > 38.12 so reject H0 (conclude regression is unstable)

Why is regression unstable?

- set dum = 1
- set dum 1960:3 1980:4 = 0
- @somlags

- FF1 1.155880251 0.443524075 2.60613
- FF2 -0.054742093 0.470867495 -0.11626
- FF3 -.943361249 0.598965882 -3.24453
- FF4 2.167587438 0.747385701 2.90023

- Alternative idea: use average fed funds rate over quarter rather than third month of quarter

- open data fed_data_avg.prn
- data(org=obs) 1959:1 2005:3 date2 fedfunds M2

- Maximum value for F-statistic (1980:4) = 2.204
- k = 17
- (2.204)(17) = 37.47
- 37.47 < 38.12 (we could accept H0)

A. Principles of forecasting
B. Hypothesis testing
C. Maximum likelihood and quasi-MLE

Suppose we make an even stronger assumption:

$$\varepsilon_t | \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_1 \sim N(0, \sigma^2)$$

Then the sample conditional log likelihood would be

$$\sum_{t=1}^{T} \log f(y_{1t} | \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p})$$

$$= -\frac{T}{2}\log(2\pi\sigma^2) - \frac{\sum_{t=1}^{T}(y_{1t} - \boldsymbol{\beta}'\mathbf{x}_t)^2}{2\sigma^2}$$

Observe: value of $\boldsymbol{\beta}$ that maximizes log likelihood is OLS estimate $\hat{\boldsymbol{\beta}}$

One option: maximize likelihood function even if we don't believe distributional assumption

= quasi-maximum likelihood estimation

MLE (or quasi-MLE) of σ^2 turns out to be

$$\hat{\sigma}^2 = T^{-1}\sum_{t=1}^{T}(y_{1t} - \hat{\boldsymbol{\beta}}'\mathbf{x}_t)^2$$

If distributional assumption is wrong (e.g., true $\varepsilon_t \sim$ Student t), then MLE is still consistent but no longer efficient

If convinced of Student t errors, better to choose $(\boldsymbol{\beta}, \sigma, \nu)$ to maximize $\sum_{t=1}^{T} q_t$

$$q_t = \log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right)$$
$$-\frac{1}{2}\log(\sigma^2\nu\pi) - \frac{\nu+1}{2}\log\left(1 + \frac{(y_t - \boldsymbol{\beta}'\mathbf{x}_t)^2}{\nu\sigma^2}\right)$$

Estimation, Forecasting, and Hypothesis Testing

A. Principles of forecasting
B. Hypothesis testing
C. Maximum likelihood and quasi-MLE
D. Viewing as a vector system

So far we've just considered forecasting y_{1t}, the first element of the vector $\mathbf{y}_t$, using

$$y_{1t} = \boldsymbol{\pi}_1'\mathbf{x}_t + \varepsilon_{1t}$$
$$\underset{(k\times 1)}{\mathbf{x}_t} = (1, \mathbf{y}_{t-1}', \mathbf{y}_{t-2}', \ldots, \mathbf{y}_{t-p}')'$$
$$k = np + 1$$

ε_{1t} = error forecasting variable 1

Obviously we could set up analogous model to forecast second variable:

$$y_{2t} = \boldsymbol{\pi}_2'\mathbf{x}_t + \varepsilon_{2t}$$
$$\underset{(k\times 1)}{\mathbf{x}_t} = (1, \mathbf{y}_{t-1}', \mathbf{y}_{t-2}', \ldots, \mathbf{y}_{t-p}')'$$

Stack these in a vector system:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\pi}_1' \\ \boldsymbol{\pi}_2' \\ \vdots \\ \boldsymbol{\pi}_n' \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix}$$

$$\underset{(n\times 1)}{\mathbf{y}_t} = \underset{(n\times k)}{\Pi'}\underset{(k\times 1)}{\mathbf{x}_t} + \underset{(n\times 1)}{\boldsymbol{\varepsilon}_t}$$

$$\Pi'\mathbf{x}_t = \begin{bmatrix} \mathbf{c} & \Phi_1 & \Phi_2 & \cdots & \Phi_p \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \\ \vdots \\ \mathbf{y}_{t-p} \end{bmatrix}$$

$$\mathbf{y}_t = \mathbf{c} + \Phi_1\mathbf{y}_{t-1} + \Phi_2\mathbf{y}_{t-2} + \cdots + \Phi_p\mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t$$

called a vector autoregression (VAR)

If all (possibly complex) scalars z satisfying

$$|\mathbf{I}_n - \Phi_1 z - \Phi_2 z^2 - \cdots - \Phi_p z^p| = 0$$

also satisfy $||z|| > 1$, then $\mathbf{y}_t$ is covariance-stationary

Example: if $p = 1$ and all eigenvalues of Φ_1 are less than unity in modulus, then VAR is covariance-stationary

Suppose we make the distributional assumption that $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Omega})$

$$\log f(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T | \mathbf{y}_0, \mathbf{y}_{-1}, \ldots, \mathbf{y}_{-p+1})$$

$$= \sum_{t=1}^{T} \log f(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p})$$

$$= \frac{-Tn}{2}\log(2\pi) - \frac{T}{2}\log|\boldsymbol{\Omega}|$$

$$- (1/2)\sum_{t=1}^{T}(\mathbf{y}_t - \Pi'\mathbf{x}_t)'\boldsymbol{\Omega}^{-1}(\mathbf{y}_t - \Pi'\mathbf{x}_t)$$

Result 1: the ith row of the MLE of Π' is given by

$$\hat{\boldsymbol{\pi}}_i' = \left(\sum_{t=1}^T y_{it}\mathbf{x}_t'\right)\left(\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}$$

Example 5: full system VAR estimation

- system
- variables gdpch inflation fedfunds mgrow
- lags 1 to 4
- det constant
- end(system)

- smpl 1960:4 2005:3
- estimate(sigma)

- VAR/System - Estimation by Least Squares
- Dependent Variable GDPCH
- Quarterly Data From 1960:04 To 2005:03

F-Tests, Dependent Variable GDPCH

Variable	F-Statistic	Signif
GDPCH	2.3850	0.0534125
INFLATION	0.8099	0.5205103
FEDFUNDS	6.2796	0.0001005
MGROW	2.4071	0.0515927

F-Tests, Dependent Variable INFLATION

Variable	F-Statistic	Signif
GDPCH	1.0176	0.3999723
INFLATION	69.0846	0.0000000
FEDFUNDS	2.7173	0.0316318
MGROW	0.2443	0.9127549

F-Tests, Dependent Variable FEDFUNDS

Variable	F-Statistic	Signif
GDPCH	4.5979	0.0015233
INFLATION	3.9117	0.0046364
FEDFUNDS	260.0742	0.0000000
MGROW	0.1242	0.9736087

F-Tests, Dependent Variable MGROW

Variable	F-Statistic	Signif
GDPCH	0.6692	0.6142184
INFLATION	1.2023	0.3119016
FEDFUNDS	0.7263	0.5751938
MGROW	16.3988	0.0000000

Result 2: the MLE of Ω is given by

$$\hat{\Omega} = T^{-1}\sum_{t=1}^T \hat{\boldsymbol{\varepsilon}}_t\hat{\boldsymbol{\varepsilon}}_t'$$

- estimate(sigma)

Covariance\Correlation Matrix of Residuals

	GDPCH	INFLATION	FEDFUNDS	MGROW
GDPCH	7.66	-0.109	0.135	0.044
INFLATION	-0.309	1.035	0.201	-0.066
FEDFUNDS	0.314	0.172	0.706	-0.449
MGROW	0.317	-0.176	-0.982	6.768

Result 3: the maximized value for the log likelihood is given by

$$-(Tn/2)[1+\log(2\pi)]-(T/2)\log|\hat{\Omega}|$$

Application: full-sample test of

H_0 : $p-1$ lags

H_A : p lags

Let

$$\hat{\Omega}(p-1) = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_t(p-1)\hat{\varepsilon}_t'(p-1)$$

$$\hat{\Omega}(p) = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_t(p)\hat{\varepsilon}_t'(p)$$

$\hat{\varepsilon}_t(s)$ = residuals from VAR with s lags

then twice likelihood ratio is

$$T[\log|\hat{\Omega}(p-1)|-\log|\hat{\Omega}(p)|] \approx \chi^2(n^2)$$

Sims small-sample correction:

$$(T-k)[\log|\hat{\Omega}(p-1)|-\log|\hat{\Omega}(p)|] \approx \chi^2(n^2)$$

Example 5 (continued): testing for lag length

- smpl 1960:4 2005:3
- estimate(sigma)
- compute logdet4 = %logdet
- compute tee = %nobs

- system
- variables gdpch inflation fedfunds mgrow
- lags 1 to 5
- det constant
- end(system)
- estimate
- compute logdet5 = %logdet

Reject H0 p=4 in favor of HA p=5

compute test_chi = tee*(logdet4 - logdet5)
- cdf chisqr test_chi 16

- Chi-Squared(16)= 36.351987 with Significance Level 0.00258480

- compute test_sims = (tee - 21.0)*(logdet4 - logdet5)
- cdf chisqr test_sims 16

- Chi-Squared(16)= 32.110922 with Significance Level 0.00967250

Accept H0 p=5 over HA p=6

- system
- variables gdpch inflation fedfunds mgrow
- lags 1 to 6
- det constant
- end(system)
- estimate
- compute logdet6 = %logdet

- Chi-Squared(16)= 28.972206 with Significance Level 0.02412421
- Chi-Squared(16)= 24.948288 with Significance Level 0.07074229

Akaike information criterion:

minimize $\log\left|\widehat{\Omega}(p)\right| + 2pn^2/T$

Schwarz criterion:

minimize $\log\left|\widehat{\Omega}(p)\right| + (pn^2/T)\log T$

Choose p=5 by AIC and p <=4 by Schwarz

- compute aic4 = logdet4 + 2*4.0*16/tee
- compute aic5 = logdet5 + 2*5.0*16/tee
- compute sic4 = logdet4 + (4.0*16/tee)*log(tee)
- compute sic5 = logdet5 + (5.0*16/tee)*log(tee)
- compute aic6 = logdet6 + 2.0*6*16/tee
- compute sic6 = logdet6 + (6.0*16/tee)*log(tee)
- display 'aic for lags 4 5 6' aic4 aic5 aic6
- disply 'schwarz for lags 4 5 6' sic4 sic5 sic6

- aic for lags 4 5 6 4.02517 4.00099 4.01781
- schwarz for lags 4 5 6 5.16044 5.42008 5.72072

Rule of thumb:

$p \geq 4$ for quarterly data

use lags $1-6$ and $11-13$ for monthly