IV. Confidence intervals for impulse-response functions

Write the VAR as

$$\mathbf{y}_t = (\mathbf{I}_n \otimes \mathbf{x}_t)\boldsymbol{\pi} + \boldsymbol{\varepsilon}_t$$

$$\underset{(n\times 1)}{\mathbf{y}_t} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$$

$$\underset{(k\times 1)}{\mathbf{x}_t} = (1, \mathbf{y}_{t-1}', \mathbf{y}_{t-2}', \ldots, \mathbf{y}_{t-p}')'$$

$$k = np + 1$$

$$\underset{(nk\times 1)}{\boldsymbol{\pi}} = (\boldsymbol{\pi}_1', \boldsymbol{\pi}_2', \ldots, \boldsymbol{\pi}_n')'$$

$$\underset{(n\times n)}{\boldsymbol{\Omega}} = E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t')$$

maximum likelihood estimates:

$$\hat{\boldsymbol{\pi}}_j = \left(\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(\sum_{t=1}^T \mathbf{x}_t y_{jt}\right)$$

$$\hat{\varepsilon}_{jt} = y_{jt} - \mathbf{x}_t' \hat{\boldsymbol{\pi}}_j$$

$$\hat{\boldsymbol{\Omega}} = T^{-1} \sum_{t=1}^T \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t'$$

$$\hat{\boldsymbol{\pi}} \approx N\left(\boldsymbol{\pi}, \boldsymbol{\Omega} \otimes \left[\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t'\right]^{-1}\right)$$

Two interpretations:

(1) classical: There is some true fixed $\boldsymbol{\pi}$. If we used this rule for estimation in a number of different samples like this one, this would be the distribution of $\hat{\boldsymbol{\pi}}$ across these samples.

$$\boldsymbol{\pi} | \boldsymbol{\Omega}, \mathbf{y}_T, \mathbf{y}_{T-1}, \ldots, \mathbf{y}_{-p+1} \sim N\left(\hat{\boldsymbol{\pi}}, \boldsymbol{\Omega} \otimes \left[\sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t'\right]^{-1}\right)$$

Two interpretations:

(2) Bayesian: The econometrician does not know the true $\boldsymbol{\pi}$ but summarizes uncertainty with a probability distribution

With either interpretation, one could summarize properties of this distribution (or functions of these parameters like the impulse-response coefficients) by simulating draws from the distribution and finding their range

Example 13: calculating one-standard deviation bands for impulse-response

Program generates draws from posterior distribution of pi and omega, calculates upper 16% and lower 16% bounds for each shock j, response i, horizon h