Answer key for the 2005 final exam (econ 220b).

1. (a) Let $x_t = (r_t, y_t)'$, $\hat{\delta} = \left(\sum x_t x_t'\right)^{-1} \sum x_t p_t$ and $\hat{z}_t = (\hat{\delta}' x_t, r_t)'$. Then we can estimate c consistently from the formula

$$\begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix} = \left(\sum \hat{z}_t \hat{z}_t'\right)^{-1} \sum \hat{z}_t q_t.$$

(b) We need to assume that $S_{xx} = \operatorname{plim} T^{-1} X'X$ and $S_{xz} = \operatorname{plim} T^{-1} X'Z$ have full rank. We can rewrite our estimator as

$$\sqrt{T} \begin{pmatrix} \hat{c} - c \\ \hat{d} - d \end{pmatrix} = \left\{T^{-1}(Z'X(X'X)^{-1}X'Z)^{-1}\right\}\left\{T^{-1/2}Z'X(X'X)^{-1}X'\mathbf{v}\right\}$$

with $z_t = (p_t, r_t)'$. Now apply the mds central limit theorem to $T^{-1/2}X'\mathbf{v}$. Then the resulting distribution for our scaled esimator is $N\left(0, \sigma_v^2 \left(S_{xz}' S_{xx}^{-1} S_{xz}\right)^{-1}\right)$.

(c) Construct the t-statistic as $\sqrt{T}\hat{c}/\sqrt{\hat{\sigma}_{11}^2}$ where $\hat{\sigma}_{11}^2$ is the top left element of the plug-in estimator of the asymptotic covariance matrix in part (b). We would reject the two-sided null hypothesis that $c = 0$ at the 5% level if the absolute value of the t-statistic is above 1.96.

2. (a)

$$X = \begin{pmatrix} 1 & x_1 \\ \vdots & \\ 1 & x_T \end{pmatrix} \qquad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix}.$$

Then $\hat{\beta} = (X'X)^{-1}X'Y$.

(b) The robust standard errors are the square roots of the diagonal elements of the matrix

$$T^{-1} \sum \hat{u}_t^2 x_t x_t'$$

where $\hat{u}_t$ is the OLS residual.

(c) We can estimate $\hat{\alpha}_3$ and $\hat{\alpha}_1$ by regressing $\log u_t^2$ on a constant and x_t, then approximate the GLS weights with $\hat{\omega}_t = \exp(\hat{\alpha}_3 - \hat{\alpha}_1 x_t)$. The feasible GLS estimator is then

$$\left[X'\hat{\Omega}^{-1}X\right]^{-1}X'\hat{\Omega}^{-1}Y$$

where $\hat{\Omega} = \text{diag}(\hat{\omega}_1, \ldots, \hat{\omega}_T)$.

(d) The standard errors are the square roots of the diagonal elements of $(X'\hat{\Omega}^{-1}X)^{-1}$.

(e) The OLS estimator is inefficient relative to a correctly specified GLS estimator and so will have a higher asymptotic variance. Using the robust standard errors does not affect this fact.

3. (a) If $U'(c) = c^{\gamma}$ then we immediately get the conditional moment restriction

$$E_t\{\beta(C_{t+1}/C_t)^{\gamma}(1 + r_t) - 1\} = 0.$$

As a consequence, the unconditional covariance of $\beta(C_{t+1}/C_t)^{\gamma}(1 + r_t) - 1$ and any time t-measurable random variable is zero.

(b) $h(\theta, w_t) = \beta(C_{t+1}/C_t)^{\gamma}(1 + r_t)x_t - x_t$ and (take derivatives with respect to β and γ)

$$\hat{D}' = \left((C_{t+1}/C_t)^{\gamma}(1 + r_t)x_t \quad \beta\log(C_{t+1}/C_t)(C_{t+1}/C_t)^{\gamma}(1 + r_t)x_t\right]\right).$$

Since $h(\theta, w_t)$ should be a martingale difference sequence (from the economic motivation earlier), we can estimate S with

$$\hat{S} = T^{-1}\sum_{t=1}^{T} h(\hat{\theta}, w_t)h(\hat{\theta}, w_t)'.$$

(c) Denote $(0.98, 0.5)$ as θ_0. A test statistic that will have chi-square distribution with two degrees of freedom under the null hypothesis that $\theta = \theta_0$ is

$$T(\hat{\theta} - \theta_0)'\hat{V}^{-1}(\hat{\theta} - \theta_0).$$

(d) Since we have more equations than parameters, we can do an overidentification test. Remember that we could construct a consistent estimate using any two (or three, or four) of the five moment conditions—one way to think of the overidentification test is that it tests whether all of those estimators give the same estimates of γ and β. The test statistic for doing this test efficiently will have a chi-square distribution with three degrees of freedom under the null hypothesis that all of our five moment conditions hold, and its formula is

$$Tg(\hat{\theta}, Y_T)'\hat{S}^{-1}g(\hat{\theta}, Y_T).$$

4. See the textbook.