

Answer key for the 2006 final exam (econ 220b).

1. See the textbook.
2. (a) $b = (X'X)^{-1}X'Y$. The OLS estimator is unbiased because the errors are strictly exogenous, and it is consistent because both the bias and variance decrease to zero in the limit.
(b) $\hat{\beta}_{GLS} = (X'V^{-1}X)^{-1}X'V^{-1}y$. The GLS estimator is also unbiased and consistent for the same reasons as the OLS estimator.
(c) If we could estimate α consistently, the feasible GLS estimator would still be consistent (unless the function V were very poorly behaved). The feasible GLS estimator will probably be biased, though. Transforming the errors (i.e. premultiplying by $\hat{V}^{-1/2}$) will remove their strict exogeneity.
3. (a)

$$\begin{aligned}\hat{\beta}_{IV} &= (X'Z)^{-1}X'(Z\beta + u) \\ &= \beta + \left(T^{-1} \sum x_t z_t'\right)^{-1} \left(T^{-1} \sum x_t' u_t\right) \\ &\rightarrow \beta\end{aligned}$$

(b)

$$\sqrt{T}(\hat{\beta}_{IV} - \beta) = \left(T^{-1} \sum x_t z_t'\right)^{-1} \left(T^{-1/2} \sum x_t' u_t\right)$$

The IV estimator is asymptotically normal because the second term is the sum of a martingale difference sequence, and $V = \sigma^2 A^{-1} B A^{-1}$.

(c) Use the test statistic

$$(R\hat{\beta}_{IV} - r)' [R\hat{\sigma}^2 \hat{A}^{-1} \hat{B} \hat{A}^{-1} R']^{-1} (R\hat{\beta}_{IV} - r)$$

where \hat{A} , \hat{B} , and $\hat{\sigma}^2$ are the plug-in estimators. Under the null hypothesis, this statistic will have a chi-square distribution with degrees

of freedom equal to the number of elements of r , so we would reject the null hypothesis if this statistic is larger than the chi-square critical value.

4. (a) In the GMM notation, $w_t = (y_t, x_t)$, $\theta = \beta$, and

$$\begin{aligned} h(\theta, w_t) &= x_t u_t \\ &= x_t (y_t - x_t' \beta). \end{aligned}$$

Since the system of equations is just-identified, we get the GMM estimator by setting $g(\theta, Y)$ to zero and solving for θ , which gives $\beta = (X'X)^{-1}X'Y$.

- (b)

$$\begin{aligned} \hat{V} &= \left[T \left\{ \sum x_t x_t' \right\} \left\{ \sum x_t x_t' \hat{u}_t^2 + \sum_{v=1}^q \left(1 - \frac{v}{q+1} \right) \left(\sum \hat{u}_{t-v} \hat{u}_t [x_t x_{t-v}' + x_{t-v} x_t] \right) \right\}^{-1} \right. \\ &\quad \left. \times \left\{ \sum x_t x_t' \right\} \right]^{-1} \\ \hat{D}' &= T^{-1} \sum x_t x_t' \\ \hat{S}' &= T^{-1} \sum x_t x_t' \hat{u}_t^2 + T^{-1} \sum_{v=1}^q \left(1 - \frac{v}{q+1} \right) \left(\sum \hat{u}_{t-v} \hat{u}_t [x_t x_{t-v}' + x_{t-v} x_t] \right) \end{aligned}$$

- (c) An appropriate standard error is the square root of the first element of the matrix in (b).