A Neoclassical Model of Unemployment and the Business Cycle

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This paper investigates a general equilibrium model of unemployment and the business cycle in which specialization of labor plays a key role. A rational expectations equilibrium with fully flexible wages and prices can exhibit unemployment in which the marginal product of employed workers exceeds the reservation wage of those who are without jobs. Workers are unemployed either because they are in the process of relocating for a better job or because they are waiting for conditions in the depressed sector to improve. Moreover, seemingly small disruptions in the supplies of primary commodities such as energy could be the source of fluctuations in aggregate employment and can exert surprisingly large effects on real output.

I. Introduction

This paper argues that specialization of labor and capital accounts for many of the features observed for unemployment and the business cycle. The theme is not new. Ricardo's (1817) chapter 19, "On Sudden Changes in the Channels of Trade," articulates many of the issues taken up in this paper. Feldstein (1975) and Jovanovic (1979, 1984) have emphasized that the nontrivial allocation problem in matching workers with jobs best suited for their characteristics is crucial to understanding unemployment, while Black (1982), Lilien (1982), and Davis (1984, 1985, 1987) suggested that the difficulties in allocating labor across different sectors may play a causal role in business cycles.

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Davis (1984, 1985) and Loungani (1985, 1986) in particular have developed the theme pursued here that energy price shocks may be a key cause of sectoral imbalance.

The current paper claims three contributions. First, I show that the role of specialization in unemployment and the business cycle can be rigorously grounded in a fully specified general equilibrium model with rationally formed expectations. Second, I suggest that the model is able to capture some of the features of what we normally think of as "involuntary" unemployment. Third, I show that large fluctuations in output could be generated by seemingly small disruptions in the supply of primary commodities such as energy.

These points address what are widely perceived to be some of the most important gaps in real business cycle theory.[1] The themes pursued here could have been explored in a sticky-price model, in which the effects I describe would be all the more dramatic. In modeling the essential rigidities in the economy exclusively in terms of specification of technology rather than through assumptions about suboptimal pricing arrangements, I hope to have called attention to the importance of the former for our understanding of unemployment and the business cycle.

The principal propagation mechanism of the business cycle explored in this paper is the possibility that an energy price increase will depress consumer purchases of energy-using goods such as automobiles.[2] The dollar value of such purchases may be large relative to the value of the energy they use. If labor were able to relocate smoothly from one sector to another, most of the lost output would be made up by gains in other sectors. On the other hand, if there are costs or delays associated with labor mobility, then the losses of one sector need not be regained by another, and the short-term aggregate loss can exceed the dollar value of the lost energy by a substantial margin. Moreover, the period of unemployment is not necessarily limited by the amount of time necessary to relocate. If there is some probability of a return to better conditions, unemployed workers may rationally choose not to relocate, even if jobs offered in other sectors pay a wage that exceeds their marginal utility of leisure.

Even though the model is market clearing, unemployed workers are unhappy about losing their jobs and may envy currently employed workers. The essential friction that accounts for this phenom-

[1] Among the important contributions to this literature are Kydland and Prescott (1982), Long and Plosser (1983), Barrow and King (1984), King and Plosser (1984), Kydland (1984, 1985), Hansen (1985), and McCallum (1986).

[2] Davis (1984) has emphasized that an energy price decrease can also increase unemployment in this model.

enon is the requirement that workers must forgo one period's wages if they wish to switch to a more prosperous sector. Subject to this technological constraint, wages and prices are perfectly flexible.

The paper is organized as follows. Section II sets out the basic model, with equilibrium formally characterized in Section III. Section IV discusses some of the advantages of this view of unemployment over alternative theories, while Section V analyzes the size of the disturbances necessary to result in such effects.

II. The Model

An individual worker may seek employment in either of two sectors. I assume that preferences and the production technology are such that the worker has no desire or opportunity to work less than the standard workweek at a given job.[3] Thus a particular worker is either employed full-time in sector 1, employed full-time in sector 2, or unemployed.

In addition to the outputs from the two sectors, there is an unproduced good in the economy with which households are exogenously endowed. The economywide supply at date t of this good is denoted $\bar{X}_t$. Households may buy or sell X to one another or to firms, and its price is determined endogenously.

The general equilibrium model that I present below has the feature that a decrease in $\bar{X}_t$ reduces the utility of workers in both sectors, though those employed in sector 1 are harder hit than those in sector 2. Changes in labor's marginal product in sector 1 could arise through either of two channels. The first might be described as a supply effect, in which $\bar{X}_t$ is an input used along with labor in the production of good 1. When firms cut back on the use of this input, labor's marginal physical product may be reduced directly.[4] The second channel might be described as a demand effect. In this case X appears as an argument of the utility function of consumers rather than an argument of the production function of firms. An increase in the cost of X might make consumers want to cut back on their purchases of good 1; for example, an oil price shock may reduce the demand for new cars.[5] The resulting decrease in the relative price of good 1 depresses labor's marginal product in sector 1 relative to sector 2. In an earlier version of this paper, I provided explicit parametric examples of both supply and demand effects and showed that in general equilibrium

[3] See Hansen (1985) for a detailed discussion of this very important assumption.

[4] For sticky-price versions of this argument, see Phelps (1978) and Mork and Hall (1980).

[5] See Bernanke (1983) for an interesting perspective on how this effect might occur.

the two are algebraically equivalent. In the current version, only the demand effects are modeled explicitly.

Suppose we were told that the endowment of X was $\bar{X}_t$, that a specified number $L_{1,t}$ of workers turn out to be employed in sector 1 (technically, $L_{1,t}$ denotes the Lebesgue measure since I model workers as a continuum rather than a discrete set), and $L_{2,t}$ workers turn out to be employed in sector 2. We can calculate what the levels of output, real wages, and relative prices must be to persuade firms to hire workers in those numbers and to persuade consumers to purchase all that would be produced. One can then ask, for that specification of the vector of relative prices, what the current-period level of utility would be for the kth individual worker under the following three possibilities: (1) the worker is employed in sector 1, (2) the worker is employed in sector 2, or (3) the worker is unemployed. Denote these levels of utility by $v_1(\bar{X}_t, L_{1,t}, L_{2,t}; k)$, $v_2(\bar{X}_t, L_{1,t}, L_{2,t}; k)$, and $v_0(\bar{X}_t, L_{1,t}, L_{2,t}; k)$. In Section III I will go on to analyze the labor supply decision the worker would want to make between options 1, 2, and 3. Hypothesizing different values for $L_{1,t}$ or $L_{2,t}$ will change the vector of relative prices and alter the choice the worker would want to make between these options. General equilibrium will then be characterized by those values of $L_{1,t}$ and $L_{2,t}$ for which the assumed number of workers ($L_{1,t}$ and $L_{2,t}$) would indeed turn out to seek employment in the respective sectors.

In the remainder of Section II, I show that a parametric specification of preferences and production technology exists for which the functions $v_1(\cdot)$, $v_2(\cdot)$, and $v_0(\cdot)$ admit the following representations:

$$v_1(\bar{X}_t, L_{1,t}, L_{2,t}; k) = \alpha(\bar{X}_t, L_{1,t}, L_{2,t}; k) + u_1(\bar{X}_t, L_{1,t}, L_{2,t}), \tag{1}$$

$$v_2(\bar{X}_t, L_{1,t}, L_{2,t}; k) = \alpha(\bar{X}_t, L_{1,t}, L_{2,t}; k) + u_2(\bar{X}_t, L_{1,t}, L_{2,t}), \tag{2}$$

$$v_0(\bar{X}_t, L_{1,t}, L_{2,t}; k) = \alpha(\bar{X}_t, L_{1,t}, L_{2,t}; k) + \bar{u}. \tag{3}$$

Note that the term $\alpha(\cdot)$ is common to all three expressions. This is the level of utility the consumer receives independently of his personal employment decision. Equations (1)–(3) claim that the single-period utility v_i ($i = 1, 2, 0$) is additively separable between this common component $\alpha(\cdot)$ and a term $u_i(\cdot)$ that is a *monotonic* function of $\bar{X}_t$, $L_{1,t}$, and $L_{2,t}$. The relative gain in utility from working in sector 1, $u_1(\cdot)$, is monotonically increasing in $\bar{X}_t$: bigger values of $\bar{X}_t$ improve the demand for sector 1 output. The function $u_1(\cdot)$ is monotonically decreasing in L_{1t}: the greater the number of other people who are already working in sector 1, the lower the real wage that worker k could receive in that sector, and so sector 1 employment becomes less attractive relative to sector 2. The function $u_1(\cdot)$ is monotonically increasing in L_{2t}: greater employment in sector 2 increases the economy's production of good 2, and this improves the terms of trade and effective real wage for sector 1. For similar reasons, the added utility from working in sector 2, $u_2(\cdot)$, is monotonically increasing in L_{1t} and decreasing in L_{2t}. The function $u_2(\cdot)$ is also monotonically increasing in $\bar{X}_t$ as a consequence of general equilibrium wealth effects, though added $\bar{X}_t$ increases $u_1(\cdot)$ by more than it increases $u_2(\cdot)$. The term $\bar{u}$ in expression (3) corresponds to the marginal utility of leisure.

The reader who is uninterested in confirming the derivation of equations (1)–(3) may choose to skip much of the remainder of this section on a first reading.

A. *Production*

Production is carried out by two representative price-taking firms. Period t output of sector 1 is governed by the production function

$$Y_{1,t} = F(L_{1,t}), \tag{4}$$

and similarly for sector 2:

$$Y_{2,t} = G(L_{2,t}). \tag{5}$$

Output of sector 1 is taken as the numeraire; $P_{2,t}$ thus denotes the price of good 2 in terms of good 1. It turns out that in equilibrium the two sectors will in general pay different wages, $W_{1,t}$ and $W_{2,t}$, again expressed in units of good 1. Both firms are assumed to act as competitive profit maximizers and so set the marginal product of labor equal to the product wage:

$$F'(L_{1,t}) = W_{1,t}, \tag{6}$$

$$G'(L_{2,t}) = \frac{W_{2,t}}{P_{2,t}}. \tag{7}$$

B. *Households*

During period t, household k consumes amounts $c_{1,t}(k)$ of good 1, $c_{2,t}(k)$ of good 2, $x_t(k)$ of the unproduced good, and $h_t(k)$ units of leisure. The worker is either employed, $h_t(k) = 0$, or unemployed, $h_t(k) = 1$, and seeks to maximize

$$E_t \sum_{\tau=t}^{\infty} \lambda^{\tau-t} U(c_{1,\tau}(k), c_{2,\tau}(k), x_\tau(k), h_\tau(k)),$$

where E_t denotes the expectation formed at date t. My parametric example employs a nested Cobb-Douglas constant elasticity of substitution utility function:

$$U(c_{1,\tau}(k), c_{2,\tau}(k), x_\tau(k), h_\tau(k)) \equiv \{[x_\tau(k)]^\rho + b[c_{1,\tau}(k)]^\rho\}^{\theta/\rho}[c_{2,\tau}(k)]^{1-\theta} + \bar{u} \cdot h_\tau(k). \quad (8)$$

In order to model complementarity between x and c_1, I specify that $\rho < 0$, implying that there is less than unit direct elasticity of substitution between x and c_1 (but unit elasticity between this composite $x - c_1$ and c_2).

The household's income during period t comes from three sources. Its labor income is $[1 - h_t(k)]W_{i,t}$, where $W_{i,t}$ is the wage paid if the household works in sector i at date t. The household also owns a share $\phi(k)$ in the total profits π_t of the two firms and a share $z(k)$ in the economywide endowment of $\bar{X}_t$.[6] Thus its single-period budget constraint is[7]

$$c_{1,\tau}(k) + P_{2,\tau}c_{2,\tau}(k) + P_{X,\tau}x_\tau(k) = [1 - h_\tau(k)]W_{i,\tau} + \phi(k)\pi_\tau + z(k)P_{X,\tau}\bar{X}_\tau, \quad \tau = t, t+1, \ldots. \quad (9)$$

There is no store of value in this economy, and thus conditional on having chosen unemployment or employment in a particular sector i at some date t, the optimal choice of $c_{1,t}(k)$, $c_{2,t}(k)$, and $x_t(k)$ is the solution to maximizing (8) subject to (9).[8] The first-order conditions for this problem call for the consumer to spend a constant fraction $(1 - \theta)$ of his income on good 2 and fractions $\theta\gamma_t$ and $\theta(1 - \gamma_t)$ on goods X and 1, where γ_t is a function of the relative price of good X:

$$P_{X,t}x_t(k) = \theta \cdot \gamma_t \cdot \{[1 - h_t(k)]W_{i,t} + \phi(k)\pi_t + z(k)P_{X,t}\bar{X}_t\}, \quad (10)$$

$$c_{1,t}(k) = \theta \cdot (1 - \gamma_t) \cdot \{[1 - h_t(k)]W_{i,t} + \phi(k)\pi_t + z(k)P_{X,t}\bar{X}_t\}, \quad (11)$$

$$P_{2,t}c_{2,t}(k) = (1 - \theta) \cdot \{[1 - h_t(k)]W_{i,t} + \phi(k)\pi_t + z(k)P_{X,t}\bar{X}_t\}, \quad (12)$$

$$\gamma_t = \frac{[x_t(k)]^\rho}{[x_t(k)]^\rho + b[c_{1,t}(k)]^\rho}. \quad (13)$$

[6] Thus for A the set of all households, $\int_A \phi(k)dk = \int_A z(k)dk = 1$.

[7] Goods Y_1, Y_2, and X are all taken to be nonstorable. A more complete general equilibrium model would make $\phi(k)$ and $z(k)$ endogenous by allowing trade in the claims on these implicit assets. Including an asset market would seem to offer little additional insight. For my parametric example, the time separability of the utility function and the linearity of utility in wealth (eq. [15] below) would seem to eliminate any role for an insurance or contingent claims market. The effects derived below are due to the specification of technology and have nothing to do with the consumption-multiplier effects of Keynesian models or with motives for intertemporal smoothing of consumption and leisure.

[8] Later I will analyze the optimal labor decisions by using these semi-reduced-form expressions. The technique is analogous to concentrating a likelihood function (see Koopmans and Hood 1953, pp. 156–57).

A little work with (10), (11), and (13) shows that

$$\gamma_t = \frac{1}{1 + b^{1/(1-\rho)} P_{X,t}^{\rho/(1-\rho)}}, \tag{14}$$

and hence the notation; γ_t is the same for all workers k.

Substituting (13), (10), and (12) into (8) yields the single-period indirect utility function

$$\frac{\gamma_t^{\theta-(\theta/\rho)} \cdot \theta^\theta \cdot (1-\theta)^{1-\theta} \cdot \{[1 - h_t(k)]W_{i,t} + \phi(k)\pi_t + z(k)P_{X,t}\overline{X}_t\}}{P_{X,t}^\theta \cdot P_{2,t}^{1-\theta}} + \bar{u} \cdot h_t(k). \tag{15}$$

C. *Goods Market Equilibrium*

Let A denote the set of all workers. Equilibrium in the markets for Y_1, Y_2, and X requires

$$\int_A c_{1,t}(k)dk = Y_{1,t},$$

$$\int_A c_{2,t}(k)dk = Y_{2,t},$$

$$\int_A x_t(k)dk = \overline{X}_t.$$

Any two of these equations determine $P_{2,t}$ and $P_{X,t}$; the third equation can then be derived from the budget constraints of households via a familiar application of Walras's law. Integrating (11) and using the first market-clearing condition along with the definition of aggregate profits ($\pi_t \equiv Y_{1,t} - W_{1,t}L_{1,t} + P_{2,t}Y_{2,t} - W_{2,t}L_{2,t}$) yields

$$Y_{1,t} = \theta(1 - \gamma_t)(Y_{1,t} + P_{2,t}Y_{2,t} + P_{X,t}\overline{X}_t). \tag{16}$$

The third market-clearing condition together with integration of (10) likewise yields

$$P_{X,t}\overline{X}_t = \theta \cdot \gamma_t \cdot (Y_{1,t} + P_{2,t}Y_{2,t} + P_{X,t}\overline{X}_t). \tag{17}$$

Solving (16) and (17) gives the equilibrium prices

$$P_{2,t} = \frac{(1-\theta)Y_{1,t}}{\theta(1-\gamma_t)Y_{2,t}}, \tag{18}$$

$$P_{X,t} = \frac{\gamma_t Y_{1,t}}{(1-\gamma_t)\overline{X}_t}. \tag{19}$$

Using (19), we can then solve (14):

$$\gamma_t = \frac{1}{1 + b(Y_{1,t}/\bar{X}_t)^{\rho}}. \tag{20}$$

Equations (4)–(7) and (18)–(20) determine the equilibrium quantities $Y_{1,t}$, $Y_{2,t}$, $W_{1,t}$, $W_{2,t}$, $P_{2,t}$, $P_{X,t}$, and γ_t as functions of $\bar{X}_t$, $L_{1,t}$, and $L_{2,t}$. The utility of worker k (expression [15]) can then likewise be solved as a function of $\bar{X}_t$, $L_{1,t}$, and $L_{2,t}$. In particular, define α to be the component of (15) that is independent of the worker's employment decision:

$$\alpha \equiv \frac{\gamma_t^{\theta-(\theta/\rho)} \cdot \theta^{\theta} \cdot (1-\theta)^{1-\theta} \cdot [\phi(k)\pi_t + z(k)P_{X,t}\bar{X}_t]}{P_{X,t}^{\theta} \cdot P_{2,t}^{1-\theta}}.$$

Solving out for $P_{X,t}$, $P_{2,t}$, γ_t, and π_t as above allows us to write $\alpha(\bar{X}_t, L_{1,t}, L_{2,t}; k)$. The single-period utility level v_1 of a worker who ends up finding employment in sector 1 at wage $W_{1,t} = F'(L_{1,t})$ is then given by equation (1), in which

$$\begin{aligned} u_1(\bar{X}_t, L_{1,t}, L_{2,t}) \equiv {} & [\gamma(\bar{X}_t, L_{1,t})]^{-\theta/\rho} \cdot \theta \cdot [1 - \gamma(\bar{X}_t, L_{1,t})] \cdot \bar{X}_t^{\theta} \\ & \times [G(L_{2,t})]^{1-\theta} \cdot \frac{F'(L_{1,t})}{F(L_{1,t})} \end{aligned} \tag{21}$$

and

$$\gamma(\bar{X}_t, L_{1,t}) \equiv \frac{1}{1 + b[F(L_{1,t})/\bar{X}_t]^{\rho}}.$$

Likewise, the utility v_2 if employed in sector 2 at wage $W_{2,t} = G'(L_{2,t}) \cdot P_{2,t}$ is given by equation (2), where

$$\begin{aligned} u_2(\bar{X}_t, L_{1,t}, L_{2,t}) \equiv {} & [\gamma(\bar{X}_t, L_{1,t})]^{-\theta/\rho} \cdot (1-\theta) \cdot \bar{X}_t^{\theta} \\ & \times [G(L_{2,t})]^{-\theta} \cdot G'(L_{2,t}). \end{aligned} \tag{22}$$

The utility if unemployed is of course given by equation (3).

For Cobb-Douglas production functions ($F(L_1) = L_1^{\eta}$ and $G(L_2) = L_2^{\beta}$), total differentiation of (20), (21), and (22) yields

$$\begin{aligned} \frac{du_1}{u_1} = {} & (\theta - \rho)\gamma \cdot \frac{d\bar{X}}{\bar{X}} + [(1-\theta)\beta] \cdot \frac{dL_2}{L_2} \\ & + \{-1 + [\theta(1-\gamma) + \rho\gamma]\eta\} \cdot \frac{dL_1}{L_1}, \end{aligned} \tag{23}$$

$$\frac{du_2}{u_2} = (\theta\gamma) \cdot \frac{d\bar{X}}{\bar{X}} + [-1 + (1-\theta)\beta] \cdot \frac{dL_2}{L_2} + [\theta\eta(1-\gamma)] \cdot \frac{dL_1}{L_1}. \tag{24}$$

Note that for $\rho < 0$, u_1 is decreasing in L_1 and increasing in L_2, whereas u_2 is increasing in L_1 and decreasing in L_2, as claimed at the beginning of this section. Likewise u_1 and u_2 are both increasing in $\bar{X}$, though more $\bar{X}$ tends to make sector 1 employment more attractive relative to sector 2. This completes the derivation of equations (1)–(3).

III. General Equilibrium

Suppose that the economywide endowment $\bar{X}_t$ takes on one of two values, M or m (with $M > m$), according to a Markov process:

$$\begin{aligned}
\text{prob}[\bar{X}_t &= M|\bar{X}_{t-1} = M] = p, \\
\text{prob}[\bar{X}_t &= m|\bar{X}_{t-1} = M] = 1 - p, \\
\text{prob}[\bar{X}_t &= M|\bar{X}_{t-1} = m] = 1 - q, \\
\text{prob}[\bar{X}_t &= m|\bar{X}_{t-1} = m] = q,
\end{aligned}$$

where prob$[A|B]$ denotes the probability of event A conditional on event B. An individual worker (k) is assumed to know these probabilities and the reduced-form functions (1)–(3). His employment decision is represented by the choice of a sequence $\{\sigma_{\tau,k}\}_{\tau=t}^{\infty}$ with $\sigma_{\tau,k} = 0, 1$, or 2 so as to maximize

$$E_t \sum_{\tau=t}^{\infty} \lambda^{\tau-t} \cdot v_{\sigma_{\tau,k}}(\bar{X}_\tau, L_{1,\tau}, L_{2,\tau}; k)$$

$$= E_t \sum_{\tau=t}^{\infty} \lambda^{\tau-t} \cdot \alpha(\bar{X}_\tau, L_{1,\tau}, L_{2,\tau}; k) + E_t \sum_{\tau=t}^{\infty} \lambda^{\tau-t} \cdot u_{\sigma_{\tau,k}}(\bar{X}_\tau, L_{1,\tau}, L_{2,\tau}).$$

The first term on the right-hand side of this expression represents the utility the worker will receive no matter what employment decision he makes, and it is irrelevant for the analysis of this section.

My specification of the nature of frictions in the labor market and the basic equilibrium methodology is adapted from Lucas and Prescott (1974). I impose a feasibility constraint similar to theirs, which I interpret as a restriction imposed by the technology of location and training.[9] A worker must precommit at date t to the sector in which he will seek employment at date $t + 1$. Once date $t + 1$ arrives, the

[9] An alternative interpretation, which would bring this account closer to that in Kydland and Prescott (1982), is that relocation of workers requires construction of new idiosyncratic physical capital that takes one period to build. The location interpretation is perhaps the most natural given my particular specification of the technology. According to my model, idiosyncratic human capital never depreciates as long as one remains committed to the same sector, and it depreciates 100 percent as soon as one switches sectors.

worker must choose between employment in that precommitted sector or unemployment. Most important, I specify that the worker can switch from one sector to another only by first going through a period of unemployment.

My approach is to conjecture a particular structure for the time series $\{L_{1,t}\}$ and $\{L_{2,t}\}$ in general equilibrium. I then show that, if relative prices in the aggregate economy were those associated with these sequences, then an individual worker who lived in such an economy would indeed optimally choose a time path $\{\sigma_{t,k}\}$ such that, when these individual paths are aggregated across all workers k, the labor supplied to each sector at each date would equal the hypothesized magnitudes $L_{1,t}$ and $L_{2,t}$.

The conjectured equilibrium is one in which an individual worker would consider switching sectors only when the aggregate endowment of $\bar{X}_t$ changes. If $\bar{X}_t = \bar{X}_{t-1}$, any worker will make the same sectoral commitment at date t as that individual made at $t - 1$. The macroeconomic equilibrium thus consists of four separate states: state 1: $\bar{X}_t = M$ and $\bar{X}_{t-1} = M$; state 2: $\bar{X}_t = m$ and $\bar{X}_{t-1} = M$; state 3: $\bar{X}_t = M$ and $\bar{X}_{t-1} = m$; state 4: $\bar{X}_t = m$ and $\bar{X}_{t-1} = m$. Each of these states ($j = 1, 2, 3, 4$) implies a particular exogenous value for $X(j)$ and equilibrium values for $L_1(j)$ and $L_2(j)$. For example, if the economy is in state 3 at date t, then $\bar{X}_t = X(3) = M$.

The magnitudes $L_1(j)$ and $L_2(j)$ for $j = 1, 2, 3, 4$ are in principle all endogenous. I have found it expositionally simplest just to posit a second class of workers who are assumed to have zero marginal utility of leisure and who lack an innate attribute necessary to work in sector 1. These individuals are thus always employed in sector 2. Let $\bar{L}_2$ denote the total number of these immobile workers and $\bar{L}_1$ the total number of mobile workers on which the entire analysis focuses. I then will choose exogenous parameters in such a way that everyone who could would want to be employed in sector 1 in state 1. It will thus turn out that $L_1(1) = \bar{L}_1$ and $L_2(1) = \bar{L}_2$; $L_1(j)$ and $L_2(j)$ for $j = 2, 3, 4$ can all be conveniently parameterized relative to $\bar{L}_1$ and $\bar{L}_2$. Assuming the existence of this second class of immobile workers also shortens the discussion of the kinds of switching strategies that have to be considered in general equilibrium. The primary role of the immobile workers is thus to simplify the notation and exposition; they have essentially nothing to do with the substantive claims made below. Throughout the paper, whenever I refer to the motives of "workers" I refer exclusively to the motives of mobile workers.

Note that, under the conjectured equilibrium, a sufficient statistic for describing the current and forecasting all future macroeconomic conditions is whether the current state (s_t) takes on the value 1, 2, 3, or 4. If in addition we know the sector ($\zeta_{t,k}$) to which the kth worker is committed for period t (as a consequence of a decision he made at $t-1$), then we can calculate the maximum expected lifetime utility for that individual:

$$J(s_t, \zeta_{t,k}) \equiv \max_{\{\sigma_{\tau,k}, \zeta_{\tau+1,k}\}_{\tau=t}^{\infty}} E_t \sum_{\tau=t}^{\infty} \lambda^{\tau-t} \cdot u_{\sigma_{\tau,k}}(\bar{X}_\tau, L_{1,\tau}, L_{2,\tau}) \qquad (25)$$

subject to $\sigma_{\tau,k} \in \{0, \zeta_{\tau,k}\}$.

Appendix A establishes that we can limit our analysis to state-dependent strategies. That is, a particular worker is following a strategy such as $\boldsymbol{\xi}(k) = (1, 0, 0, 2)$, which notation conveys that worker k is employed in sector 1 whenever the macroeconomy is in state 1, employed in sector 2 whenever the macroeconomy is in state 4, and switching between the two in states 2 and 3.

We can then arrive at closed-form expressions for equation (25). Let $p_{ij}(n)$ denote the probability that n periods from today the economy will be in state j, given that today's state is i. Let $\mathbf{P}(n)$ be the 4×4 matrix whose (i, j)th element is $p_{ij}(n)$. Thus $\mathbf{P}(0)$ is the identity matrix and

$$\mathbf{P}(1) = \begin{bmatrix} p & (1-p) & 0 & 0 \\ 0 & 0 & (1-q) & q \\ p & (1-p) & 0 & 0 \\ 0 & 0 & (1-q) & q \end{bmatrix}. \qquad (26)$$

Suppose that the economy is currently in state $s_t = i$. Then the expected returns to an arbitrary state-dependent strategy $\boldsymbol{\xi}$ can be written as

$$\begin{aligned} V(\boldsymbol{\xi}, s_t)\big|_{s_t=i} &= \sum_{j=1}^{4} \sum_{n=0}^{\infty} p_{ij}(n) \cdot \lambda^n \cdot u_{\xi_j}(X(j), L_1(j), L_2(j)) \\ &= \sum_{j=1}^{4} u_{\xi_j}(X(j), L_1(j), L_2(j)) \sum_{n=0}^{\infty} p_{ij}(n) \cdot \lambda^n, \end{aligned} \qquad (27)$$

where ξ_j denotes the jth element of the vector $\boldsymbol{\xi}$. Thus in steady state,[10]

$$J(s_t, \zeta_{t,k}) = \max V(\boldsymbol{\xi}, s_t)$$

subject to $\xi_{s_t} \in \{0, \zeta_{\tau,k}\}$. Evaluation of (27) requires an expression for $\Sigma_{n=0}^{\infty} p_{ij}(n) \cdot \lambda^n$. Recall (e.g., Chiang 1980, p. 109) that, for a Markov process, $\mathbf{P}(n) = [\mathbf{P}(1)]^n$, and so since $0 \leq \lambda < 1$,

$$\mathbf{I} + \lambda \cdot \mathbf{P}(1) + \lambda^2 \cdot \mathbf{P}(2) + \lambda^3 \cdot \mathbf{P}(3) + \ldots = [\mathbf{I} - \lambda \cdot \mathbf{P}(1)]^{-1}.$$

[10] That is, we consider a worker for whom $\zeta_{t,k}$ represents a decision previously made as part of an optimal state-dependent strategy.

From (26) one calculates

$$[\mathbf{I} - \lambda \cdot \mathbf{P}(1)]^{-1} = \Delta^{-1} \cdot$$

$$\begin{bmatrix} [\Delta + \lambda p(1 - \lambda q)] & [\lambda(1 - p)(1 - \lambda q)] & [\lambda^2(1 - p)(1 - q)] & [\lambda^2 q(1 - p)] \\ [\lambda^2 p(1 - q)] & [(1 - \lambda p)(1 - \lambda q)] & [\lambda(1 - q)(1 - \lambda p)] & [\lambda q(1 - \lambda p)] \\ [\lambda p(1 - \lambda q)] & [\lambda(1 - p)(1 - \lambda q)] & [(1 - \lambda p)(1 - \lambda q)] & [\lambda^2 q(1 - p)] \\ [\lambda^2 p(1 - q)] & [\lambda^2(1 - p)(1 - q)] & [\lambda(1 - q)(1 - \lambda p)] & [\Delta + \lambda q(1 - \lambda p)] \end{bmatrix}, \tag{28}$$

where $\Delta \equiv (1 - \lambda)(1 + \lambda - \lambda p - \lambda q)$. Thus $\Sigma_{n=0}^{\infty} p_{ij}(n) \cdot \lambda^n$ is the (i, j)th element of (28).

I now seek to characterize those state-dependent strategies $\boldsymbol{\xi}$ that will be adopted in equilibrium. Suppose that the economy is currently in state $s_t = 3$, and consider a mobile worker who is committed to sector 2 ($\zeta_{t,k} = 2$). (Such a worker may be hypothetical; our ultimate purpose is to see whether someone would be willing to follow a strategy that put him in sector 2 in state 3.) The strategies $\boldsymbol{\xi}' \equiv (2, 2, 2, 2)$ and $\boldsymbol{\xi}'' \equiv (1, 0, 0, 2)$ are both available to that worker. If

$$V(\boldsymbol{\xi}', 3) < V(\boldsymbol{\xi}'', 3), \tag{29}$$

then no worker would choose $\boldsymbol{\xi}'$; everyone who can will switch from sector 2 to sector 1 each time the economy is in state 3. We can calculate a closed-form expression for (29) by using equations (27) and (28):

$$\begin{aligned} V(\boldsymbol{\xi}'', 3) - V(\boldsymbol{\xi}', 3) = \Delta^{-1} \cdot \{&[\lambda p(1 - \lambda q)][u_1(X(1), L_1(1), L_2(1)) \\ &- u_2(X(1), L_1(1), L_2(1))] \\ &+ [\lambda(1 - p)(1 - \lambda q)][\bar{u} - u_2(X(2), L_1(2), L_2(2))] \\ &+ [(1 - \lambda p)(1 - \lambda q)][\bar{u} - u_2(X(3), L_1(3), L_2(3))]\}. \end{aligned} \tag{30}$$

Recall that $u_1(\cdot)$ is decreasing in L_1 and increasing in L_2, whereas $u_2(\cdot)$ is increasing in L_1 and decreasing in L_2. Consider, therefore, the consequences of the following inequality:

$$\begin{aligned} \lambda p[u_1(M, \bar{L}_1, \bar{L}_2) - u_2(M, \bar{L}_1, \bar{L}_2)] + \lambda(1 - p)[\bar{u} - u_2(m, \bar{L}_1, \bar{L}_2)] \\ + (1 - \lambda p)[\bar{u} - u_2(M, \bar{L}_1, \bar{L}_2)] > 0. \end{aligned} \tag{31}$$

If (31) holds, then (30) must be positive for all $L_1(j) \leq \bar{L}_1$ and all $L_2(j) \geq \bar{L}_2$. Thus condition (31) is a characterization of the exogenous parameters that is sufficient to ensure that no mobile worker would ever choose to be continuously employed in sector 2 in equilibrium.

One can further calculate that the strategy (1, 0, 0, 0) will dominate (0, 0, 0, 0) for all values of $L_1(1) \leq \bar{L}_1$ and $L_2(1) \geq \bar{L}_2$ provided that

$$u_1(M, \bar{L}_1, \bar{L}_2) - \bar{u} > 0. \tag{32}$$

Finally, I assume a condition implying that in state 1, sector 1 employment is strictly preferred to sector 2 for all values of $L_1(1) \leq \bar{L}_1$ and $L_2(1) \geq \bar{L}_2$:

$$u_1(M, \bar{L}_1, \bar{L}_2) - u_2(M, \bar{L}_1, \bar{L}_2) > 0. \tag{33}$$

Throughout the following analysis I assume that parameters are such that (31)–(33) hold along with $M > m$. I show in Appendix B that these assumptions ensure that all mobile workers are employed in sector 1 in state 1: $L_1(1) = \bar{L}_1$ and $L_2(1) = \bar{L}_2$.

The feasible equilibria can then be separated into four cases. In case A, unemployment arises because workers in the depressed sector are waiting for conditions to improve. In case B, unemployment arises because workers choose to switch sectors. Case C involves permanent full employment, while case D can exhibit a mixture of both waiting-time and sector-switching unemployment.

Case A

Suppose that, in addition to (31)–(33), the following inequalities hold:

$$u_1(m, \bar{L}_1, \bar{L}_2) - \bar{u} < 0, \tag{34}$$

$$\lambda q[u_2(m, \bar{L}_1, \bar{L}_2) - u_1(m, \bar{L}_1, \bar{L}_2)] + \lambda(1 - q)[\bar{u} - u_1(M, \bar{L}_1, \bar{L}_2)] + (1 - \lambda q)[\bar{u} - u_1(m, \bar{L}_1, \bar{L}_2)] < 0. \tag{35}$$

Condition (35) is the mirror image of (31) and implies that when the economy is currently in state 2, the strategy (1, 1, 1, 1) dominates (1, 0, 0, 2) for all $L_1(j) \leq \bar{L}_1$ and all $L_2(j) \geq \bar{L}_2$. Thus the strategy (1, 0, 0, 2) would never be adopted by a rational worker in this equilibrium. Strategy (2, 0, 0, 1) is of course dominated by (1, 0, 0, 1) by (33). With switching between sectors thus ruled out, in the case A equilibrium the magnitudes for state 3 will be the same as in state 1, just as those for state 2 will be the same as in 4.

Nor can the equilibrium call for full employment in all states. If $L_1(2)$ and $L_1(4)$ were equal to $\bar{L}_1$, then (34) implies that every mobile worker would want to quit sector 1 in states 2 and 4. Employment must fall in these states until it equals the value L_1^A defined implicitly by

$$u_1(m, L_1^A, \bar{L}_2) = \bar{u}. \tag{36}$$

Since $u_1(m, \bar{L}_1, \bar{L}_2)$ is continuous and monotonically decreasing in L_1 with $u_1(m, 0, \bar{L}_2) = \infty$, inequality (34) implies that a unique value of L_1^A exists satisfying (36), and this is the value at which workers are just indifferent between (1, 1, 1, 1) and (1, 0, 1, 0).

We are thus led to the following candidate equilibrium: some mobile workers are following the strategy (1, 1, 1, 1) while others are following (1, 0, 1, 0). As a result, $L_2(j) = \bar{L}_2$ for $j = 1, 2, 3, 4$, $L_1(1) = L_1(3) = \bar{L}_1$, and $L_1(2) = L_1(4) = L_1^A$. Workers are indifferent between strategies (1, 1, 1, 1) and (1, 0, 1, 0) in all states of nature and strictly prefer these to (1, 0, 0, 2).

This equilibrium looks just like the classical analysis of unemployment that arises when the marginal product of labor falls below the marginal utility of leisure, with one important exception: nothing about the structure of the equilibrium precludes ex post regret. It is perfectly possible that the unemployed workers of sector 1 envy those who are still employed in sector 2 in the sense that $\bar{u} < u_2(X(2), L_1(2), L_2(2))$ or even perhaps $J(s_t = 2, \zeta_{t,k} = 1) < J(s_t = 2, \zeta_{t,k} = 2)$. If an unemployed worker were able to trade places instantly and costlessly with one of the employed in another sector, he might well wish to do so. Given the real costs of doing so, however, he chooses not to, and it is in this sense that the labor market clears in this model even though the marginal utility of leisure for an unemployed worker may be less than the marginal product of labor for some of those who remain working.

Case B

Suppose instead that inequalities (34) and (35) were reversed:

$$u_1(m, \bar{L}_1, \bar{L}_2) - \bar{u} > 0, \tag{37}$$

$$\lambda q[u_2(m, \bar{L}_1, \bar{L}_2) - u_1(m, \bar{L}_1, \bar{L}_2)] + \lambda(1 - q)[\bar{u} - u_1(M, \bar{L}_1, \bar{L}_2)] + (1 - \lambda q)[\bar{u} - u_1(m, \bar{L}_1, \bar{L}_2)] > 0. \tag{38}$$

Under these conditions, if we again (wrongly) hypothesized that $L_1(j) = \bar{L}_1$ and $L_2(j) = \bar{L}_2$ in all states j, all mobile workers would want to quit sector 1 in state 2 because, from (38), (1, 0, 0, 2) would look better than (1, 1, 1, 1) whenever the economy was in state 2. The level of employment L_1 in states 2–4 thus must fall (and the level of employment L_2 in state 4 correspondingly rise) sufficiently far to raise the attractiveness of (1, 1, 1, 1) and reduce that of (1, 0, 0, 2) until, conditional on the economy's being in state 2 (when the choice between these two strategies is made), workers are indifferent. Thus the equilibrium consists of the strategy (1, 1, 1, 1) being adopted by some workers and (1, 0, 0, 2) adopted by others, with $L_1(1) = \bar{L}_1$, $L_1(2) = L_1(3) = L_1(4) = L_1^B$, $L_2(1) = L_2(2) = L_2(3) = \bar{L}_2$, and $L_2(4) = \bar{L}_1 + \bar{L}_2 - L_1^B$. Here L_1^B is defined by

$$\lambda q[u_2(m, L_1^B, \bar{L}_1 + \bar{L}_2 - L_1^B) - u_1(m, L_1^B, \bar{L}_1 + \bar{L}_2 - L_1^B)] + \lambda(1 - q)[\bar{u} - u_1(M, L_1^B, \bar{L}_2)] + (1 - \lambda q)[\bar{u} - u_1(m, L_1^B, \bar{L}_2)] = 0. \tag{39}$$

Again inequality (38) together with our assumptions about $u_1(\cdot)$ and $u_2(\cdot)$ ensures the existence of a unique value L_1^B satisfying (39).

Considering again the reference case $L_1(j) = \overline{L}_1$ and $L_2(j) = \overline{L}_2$ for all states j, (37) would imply that no one who remained committed to sector 1 would ever choose unemployment in the reference case. Since decreasing L_1 relative to $\overline{L}_1$ or increasing L_2 relative to $\overline{L}_2$ only increases the superiority of sector 1 jobs over unemployment, no one who remains committed to sector 1 is ever unemployed in this equilibrium.

Note again that even though the labor market is in equilibrium, the possibility of ex post regret arises. Those workers following (1, 0, 0, 2) are relatively better off in state 4 and worse off in state 3 than those following (1, 1, 1, 1). Nevertheless, no worker has an incentive to make any decision differently because when the choice between these strategies was made (in state 2) the worker was just indifferent.

Case C

Consider next

$$u_1(m, \overline{L}_1, \overline{L}_2) - \bar{u} > 0, \tag{40}$$

$$\begin{aligned}\lambda q[u_2(m, \overline{L}_1, \overline{L}_2) - u_1(m, \overline{L}_1, \overline{L}_2)] + \lambda(1 - q)[\bar{u} - u_1(M, \overline{L}_1, \overline{L}_2)]\\ + (1 - \lambda q)[\bar{u} - u_1(m, \overline{L}_1, \overline{L}_2)] < 0.\end{aligned} \tag{41}$$

These conditions turn out to imply a full-employment equilibrium: all workers choose (1, 1, 1, 1) with $L_1(j) = \overline{L}_1$ and $L_2(j) = \overline{L}_2$ for all j. Inequality (40) implies that no worker who was permanently committed to sector 1 would ever choose unemployment, while (41) implies that (1, 1, 1, 1) dominates (1, 0, 0, 2).

Case D

The final possibility is

$$u_1(m, \overline{L}_1, \overline{L}_2) - \bar{u} < 0,$$

$$\begin{aligned}\lambda q[u_2(m, \overline{L}_1, \overline{L}_2) - u_1(m, \overline{L}_1, \overline{L}_2)] + \lambda(1 - q)[\bar{u} - u_1(M, \overline{L}_1, \overline{L}_2)]\\ + (1 - \lambda q)[\bar{u} - u_1(m, \overline{L}_1, \overline{L}_2)] > 0.\end{aligned}$$

This turns out to be the most complicated case, with a variety of mixed employment-unemployment equilibria possible depending on additional specification of parameters. Since the qualitative behavior of the system and the economic sources of unemployment arising in case D are essentially the same as in cases A and B, I omit a detailed catalog of the possibilities here.

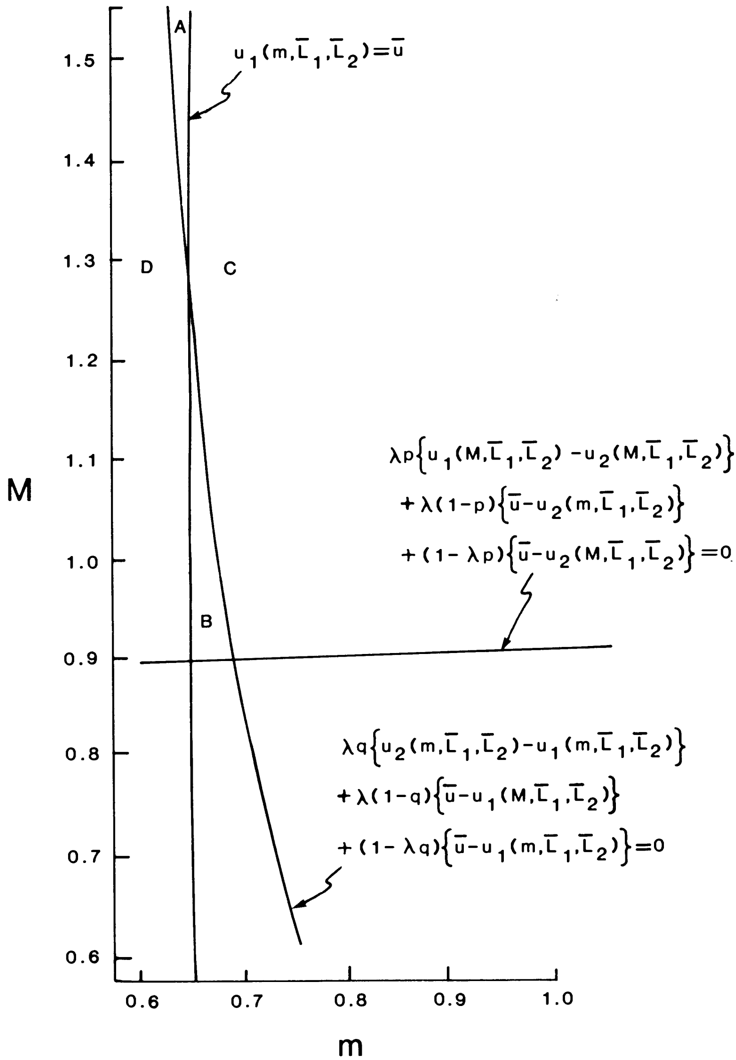

FIG. 1.—Regions A, B, C, and D plotted as functions of high (M) and low (m) levels of endowment of X.

Figure 1 illustrates cases A–D as different regions in (M, m) space. The figure was drawn for the following parameter values. Immobile workers account for 80 percent of the labor force ($\bar{L}_1 = 1$, $\bar{L}_2 = 4$), while sector 2 output constitutes 70 percent of GNP ($\theta = 0.3$). Choosing $b = 2$ implies that when $\bar{X} = 1.0$ and $L_1 = \bar{L}_1$ and $L_2 = \bar{L}_2$, the factor X has a dollar share $\gamma \cdot \theta = 0.1$ of total GNP. An important dependence of purchases of good 1 on X is reflected in the choice $\rho =$

-1.5. I further used $\lambda = 0.95$, $\eta = \beta = 0.7$, and $p = q = 0.8$. The marginal value of leisure was set equal to the utility level for workers employed in sector 1 when $X = 0.65$ ($\bar{u} = u_1(0.65, 1, 4)$). These parameter values generate all four cases A–D for relatively modest variation in X.[11]

IV. On the Nature of Unemployment

All markets clear in this model. There is thus a sense in which any unemployment that arises in this model is strictly voluntary. One could of course explore the kinds of phenomena studied here in a sticky-price framework, though I have not attempted that exercise. Instead I would like to discuss in this section the extent to which the neoclassical view of the labor market explored here might be consistent with other widely held perceptions about the nature of aggregate fluctuations in unemployment.[12]

One important stumbling block in accepting a theory of voluntary unemployment is its implication in the minds of many economists that the worker in some sense enjoys or prefers being unemployed. The neoclassical position has sometimes been parodied as explaining recessions through "a contagious attack of laziness" or ridiculed on the grounds that if recessions represent increased consumption of leisure, we should see booming sales of recreational vehicles during hard times. Such criticisms do not apply to the model presented here, for it is quite clear that the worker views the transition from state M, in which he is certain to be employed, to state m, in which he may well lose his job, as distinctly undesirable, a development that indeed may leave him quite miserable. Of course the worker would prefer to have as high a marginal product as he enjoyed in some earlier state. Unemployment in this model is not desired by the worker, but it is the best he can do with unfortunate circumstances. Unemployment is clearly bad, just not as bad as shining shoes.

Moreover, I noted above that the unemployment equilibria examined in this paper also allow the possibility of ex post regret. In the case A equilibrium, for example, workers following the (1, 0, 1, 0) strategy will be better off than workers in sector 2 in states 1 and 3 but

[11] The reader should not infer from the narrowness of regions A and B in fig. 1 that unemployment of type A or B requires a delicate knife-edge condition. By choosing $\bar{u}$ to equal $u_1(0.60, \bar{L}_1, \bar{L}_2)$ instead of $u_1(0.65, \bar{L}_1, \bar{L}_2)$ as drawn, region B becomes much bigger; by choosing $\bar{u} = u_1(0.70, \bar{L}_1, \bar{L}_2)$, A would grow. The parameters were chosen to show both regions for relatively modest variation in X. Recall further that region D exhibits the same sort of unemployment equilibria as regions A and B.

[12] My thinking on this point has been strongly influenced by Lucas's (1978) eloquent essay.

possibly worse off in states 2 and 4. Depending on how the world turns out, the unemployed can surely end up envying those who earlier received the training necessary to have the highest marginal product in the current state of the world. The unemployment is nevertheless voluntary in the sense that at the time such commitments had to be made, the path the worker actually chose was rationally expected to yield the highest utility.

I have modeled this envy as taking place across sectors; an unemployed worker committed to sector 1 would not envy a worker currently employed in sector 1 in my model. This would seem to be largely a matter of how the model was set up: if different workers at the same firm have different firm-specific skills or attributes and different histories of employment with that firm, a worker cannot ex post costlessly trade his attributes for those of another, and the possibility of envy or regret clearly still arises.

Another major stumbling block in accepting a theory of voluntary unemployment is the identification of variables that could plausibly cause a sufficiently large decrease in labor's marginal product. The model considered here highlights that it is not necessary to identify an aggregate phenomenon that depresses the marginal product of all workers below some reservation level. Instead, any event that hits one sector sufficiently hard will, at least in the short run, lead to an increase in the aggregate unemployment rate.

The "waiting-time" unemployment of case A is also consistent with empirical evidence suggesting that a substantial portion of the unemployed are eventually rehired at their old jobs (see Feldstein 1975; Lilien 1980; Katz 1986; Murphy and Topel 1987). Note that the expected duration of such unemployment is given by

$$(1 - q)(1 + 2q + 3q^2 + 4q^3 + \ldots) = \frac{1}{1 - q}.$$

Nothing in the rational expectations structure of the model prevents this from being quite a long period of time. The duration of type B unemployment in my model is given by the training or relocating delay, which again could plausibly be quite substantial.[13]

V. On the Sources of Business Fluctuations

In earlier research I reported a statistically significant correlation between oil price shocks and economic recessions in postwar U.S. data.

[13] Topel and Weiss (in press) argued that if unemployed workers are furthermore waiting for the resolution of uncertainty between states 1 and 2, this could add a significant additional source of unemployment duration. See also n. 9.

These findings have been largely corroborated with evidence from other data sets by Burbidge and Harrison (1984), Davis (1984, 1985), Loungani (1985, 1986), Santini (1985), and Gisser and Goodwin (1986). Postwar oil shocks could not have been predicted statistically on the basis of previous behavior of macroeconomic aggregates (Hamilton 1983) and moreover can convincingly be traced to specific exogenous historical events (Hamilton 1985). Such evidence makes it difficult to reject the historical correlation as entirely spurious.

Let us first ask how large a change between M and m is necessary for our model economy to exhibit an unemployment equilibrium of type A. Use (23) to linearize $u_1(m, \bar{L}_1, \bar{L}_2)$ around $\log m = \log M$:

$$u_1(m, \bar{L}_1, \bar{L}_2) \approx u_1(M, \bar{L}_1, \bar{L}_2) + u_1(M, \bar{L}_1, \bar{L}_2) \cdot (\theta - \rho) \cdot \gamma(M, \bar{L}_1) \cdot (\log m - \log M).$$

Thus a first-order approximation to (34) reveals that type A unemployment will arise when

$$\log M - \log m > \frac{-\bar{u} + u_1(M, \bar{L}_1, \bar{L}_2)}{(\theta - \rho) \cdot \gamma(M, \bar{L}_1) \cdot u_1(M, \bar{L}_1, \bar{L}_2)}. \tag{42}$$

Any drop in X of a magnitude greater than (42) will generate unemployment of this type. Once the economy is in region A, one can find the contribution of the decrease from M to m at the margin by differentiating (36) and evaluating at that value m^A where $\bar{u} = u_1(m^A, \bar{L}_1, \bar{L}_2)$:

$$\frac{d \log L_1^A}{d \log m} = \frac{(\theta - \rho) \cdot \gamma(m^A, \bar{L}_1)}{1 - \theta\eta[1 - \gamma(m^A, \bar{L}_1)] - \rho\eta \cdot \gamma(m^A, \bar{L}_1)}. \tag{43}$$

Similarly, for case B a linearization of (38) around $\log m = \log M$ shows that the drop from M to m is sufficient to generate type B unemployment when

$$\log M - \log m > \frac{(1 + \lambda - 2\lambda q)\bar{u} - (1 + \lambda - \lambda q)u_1(M, \bar{L}_1, \bar{L}_2) + (\lambda q)u_2(M, \bar{L}_1, \bar{L}_2)}{[(\lambda q\theta)u_2(M, \bar{L}_1, \bar{L}_2) - (\theta - \rho)u_1(M, \bar{L}_1, \bar{L}_2)]\gamma(M, \bar{L}_1)}. \tag{44}$$

Define m^B to be that value of m for which $L_1^B = \bar{L}_1$ in equation (39). Then the contribution of m at the case B margin would be given by

$$\frac{d \log L_1^B}{d \log m} = \{[-(\lambda\theta)u_2(m^B, \bar{L}_1, \bar{L}_2) + (\theta - \rho)u_1(m^B, \bar{L}_1, \bar{L}_2)] \cdot \gamma(m^B, \bar{L}_1)\} \div \left(\lambda \cdot u_2(m^B, \bar{L}_1, \bar{L}_2) \cdot \left\{\theta\eta[1 - \gamma(m^B, \bar{L}_1)] + \left(\frac{\bar{L}_1}{\bar{L}_2}\right)[1 - \beta(1 - \theta)]\right\}\right.$$

$$+ u_1(m^B, \bar{L}_1, \bar{L}_2) \cdot \left\{1 - \theta\eta[1 - \gamma(m^B, \bar{L}_1)] \right. \tag{45}$$

$$\left. - \rho\eta \cdot \gamma(m^B, \bar{L}_1) + \lambda\left(\frac{\bar{L}_1}{\bar{L}_2}\right)\beta(1 - \theta)\right\}$$

$$\left. + \lambda(1 - q) \cdot u_1(M, \bar{L}_1, \bar{L}_2) \cdot \{1 - \theta\eta[1 - \gamma(M, \bar{L}_1)] - \rho\eta \cdot \gamma(M, \bar{L}_1)\}\right).$$

From equation (19), $\gamma = P_X X/(Y_1 + P_X X)$ denotes the dollar value of energy as a share of $Y_1 + P_X X$, and $\gamma\theta$ denotes the share of energy in total GNP ($= P_X X/[Y_l + P_2 Y_2 + P_X X]$). Expressions (42)–(45) depend in part on the factor share γ. For a smaller value of γ, a larger drop in X is necessary to generate unemployment, and the contribution of m at the margin is also smaller the lower the value of γ. However, the effect also depends on ρ, a large absolute value of which would arise when purchases of Y_1 are closely tied to X. From (42) and (44), the larger the absolute value of ρ, the smaller is the change in X necessary to produce unemployment, and from (43) and (45), the larger would be the drop in L_1^A or L_1^B at the margin. Indeed, consider the limiting case as $\rho \rightarrow -\infty$. The utility function (8) then becomes

$$\{\min[x_t(k), c_{1,t}(k)]\}^{\theta}[c_{2,t}(k)]^{1-\theta} + \bar{u} \cdot h_t(k),$$

and each consumer will choose $x_t(k) = c_{1,t}(k)$ with $\bar{X}_t = Y_{1,t}$ in equilibrium.[14] For the slightest decrease in m below M, γ would increase to one if L_1 stayed at $\bar{L}_1$, and so $u_1(m, \bar{L}_1, \bar{L}_2) = 0$. This means that (34) would hold, and unemployment would result from the slightest decrease in m below M no matter what the value of γ. Equally clearly, in the limit as $\rho \rightarrow -\infty$, equilibrium would require employment in state 2 to equal that value L_1^* satisfying $F(L_1^*) = m$ from which

$$\frac{d \log L_1^*}{d \log m} = \frac{1}{\eta}$$

at the margin, again no matter how small γ.

The total effect on the economy is thus limited not by γ, the dollar share of energy, but rather by θ, the dollar share of products whose use depends critically on energy. The intuition for this result is straightforward. If X is truly indispensable to the consumer in being able to use good Y_1 ($\rho = -\infty$), then a 10 percent reduction in X will require a 10 percent reduction in Y_1 in equilibrium. Since $Y_1 = L_1^{\eta}$, this means a $(1/\eta) \cdot 10$ percent reduction in sector 1 employment. Economists usually think of the associated displaced workers as being paid their marginal product both in the affected sector and in alternative jobs; hence the dollar value of output lost by sector 1 is supposed

[14] Obviously to include this case we must normalize units so that $M = F(\bar{L}_1)$; see Arrow et al. (1961, p. 231) on this point.

to be made up by an equivalent dollar value increase elsewhere as the workers shift to alternative jobs. However, if significant costs or delays are associated with finding new jobs, then the increase in Y_2 may follow the drop in Y_1 only after a significant lag. Indeed, it may be the case that workers perceive it not to be in their interests to look for employment in sector 2 at all, instead choosing to remain unemployed in the hopes of a return to prosperity for sector 1. Thus the difficulties in relocating specialized labor could explain why seemingly small supply disruptions can have fairly large effects on the economy as a whole.

Appendix A

Here I show that if the macroeconomy follows the four-state Markov process described in cases A, B, and C, then any individual worker's optimal strategy consists of a sector to which he will always be committed whenever the economy is in state 1 and a second sector whenever the economy is in state 4; if these sectors are different, the worker is of course switching between the two in states 2 and 3.

The value function (25) is characterized by the recursion

$$
\begin{aligned}
&J(s_t, \zeta_{t,k}) = \\
&\max\Bigg\{\Bigg[u_{\zeta_{t,k}}(X(s_t), L_1(s_t), L_2(s_t)) + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|s_t]\cdot J(s_{t+1} = j, \zeta_{t+1,k} = \zeta_{t,k})\Bigg], \\
&\qquad\Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|s_t]\cdot J(s_{t+1} = j, \zeta_{t+1,k} = \zeta_{t,k})\Bigg], \qquad\qquad \text{(A1)} \\
&\qquad\Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|s_t]\cdot J(s_{t+1} = j, \zeta_{t+1,k} \neq \zeta_{t,k})\Bigg]\Bigg\}.
\end{aligned}
$$

OBSERVATION 1. Suppose that worker k chooses to shift from sector ζ to sector ζ' at some date when the technology state was M (alternatively, m). Then the worker would never choose to shift from ζ' back to ζ at any future date when the technology state was again M (alternatively, m).

Proof. We are told that the worker chose to shift to ζ' in some period t with $\overline{X}_t = M$, say. The worker evidently preferred this to remaining committed to ζ and just taking a period of unemployment:

$$
\begin{aligned}
&\Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|\overline{X}_t = M]\cdot J(s_{t+1} = j, \zeta')\Bigg] \\
&\geq \Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|\overline{X}_t = M]\cdot J(s_{t+1} = j, \zeta)\Bigg]. \qquad \text{(A2)}
\end{aligned}
$$

But from (26), the transition probabilities are the same in any future period t' for which $\overline{X}_{t'} = M$ as they were in t. Replacing t in expression (A2) with t' reveals that the individual would not want to cycle back to the original sector ζ for period t'; he is at least as well off remaining committed to sector ζ' and just taking a period of unemployment. Q.E.D.

OBSERVATION 2. No individual will want to shift from sector 1 to sector 2 during state 4.

Proof. Consider two cases. (a) When faced with $s_t = 2$ and $\zeta_{t,k} = 1$, the worker chooses to shift from sector 1 to sector 2. Then from observation 1, the worker will never choose to shift from 2 to 1 during any subsequent state 4. But since any given state 4 must be preceded by either state 2 or state 4, the worker under case *a* would never be in sector 1 in state 4; thus for this case the observation holds trivially. (*b*) When faced with $s_t = 2$ and $\zeta_{t,k} = 1$, the worker chooses not to shift from sector 1 to sector 2. From this we can infer that

$$\max\Bigg\{\Bigg[u_1(X(2), L_1(2), L_2(2)) + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|s_t = 2]\cdot J(s_{t+1} = j, \zeta_{t+1,k} = 1)\Bigg], \Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|s_t = 2]\cdot J(s_{t+1} = j, \zeta_{t+1,k} = 1)\Bigg]\Bigg\}$$
$$\geq \Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t+1} = j|s_t = 2]\cdot J(s_{t+1} = j, \zeta_{t+1,k} = 2)\Bigg].$$

Recall that for equilibria A–C, $L_1(2) = L_1(4)$ and $L_2(2) \leq L_2(4)$. Also $X(2) = X(4)$. Thus $u_1(X(2), L_1(2), L_2(2)) \leq u_1(X(4), L_1(4), L_2(4))$. It follows from $P[s_{t+1} = j|s_t = 2] = P[s_{t'+1} = j|s_{t'} = 4]$ that for any date t' in which the economy is in state 4,

$$\max\Bigg\{\Bigg[u_1(X(4), L_1(4), L_2(4)) + \lambda\sum_{j=1}^{4} P[s_{t'+1} = j|s_{t'} = 4]\cdot J(s_{t'+1} = j, \zeta_{t'+1,k} = 1)\Bigg], \Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t'+1} = j|s_{t'} = 4]\cdot J(s_{t'+1} = j, \zeta_{t'+1,k} = 1)\Bigg]\Bigg\}$$
$$\geq \Bigg[\bar{u} + \lambda\sum_{j=1}^{4} P[s_{t'+1} = j|s_{t'} = 4]\cdot J(s_{t'+1} = j, \zeta_{t'+1,k} = 2)\Bigg],$$

which shows that if the worker did not shift in t, he would not shift in t'. Q.E.D.

OBSERVATION 3. No individual would want to shift from sector 2 to sector 1 during state 1.

Proof. The proof precisely parallels that of observation 2, using the fact that $u_2(X(3), L_1(3), L_2(3)) \leq u_2(X(1), L_1(1), L_2(1))$. Q.E.D.

Observations 2 and 3 rule out the most interesting switches that might be hypothesized to occur during states 1 and 4. For completeness, consider the last two cases: (*a*) someone who switches from sector 1 to sector 2 in state 1 (when sector 2 is in fact the less desirable of the two) and (*b*) someone who switches from sector 2 to sector 1 in state 4 (again switching against the natural ordering). These can be ruled out as well. The proof is a bit involved and unilluminating, and for this reason I offer only a very brief sketch for the case of equilibrium B. In this equilibrium, $u_2(X(4), L_1(4), L_2(4)) > u_1(X(4), L_1(4), L_2(4))$, and $u_i(X(j), L_1(j), L_2(j)) > \bar{u}$ for $j = 1, 2, 3, 4$ and $i = 1, 2$. From these one can establish that any hypothesized non-state-dependent strategy associated with a or b is dominated by the state-dependent (but infeasible) strategy (2, 2, 1, 1). One can then use (31) to show that $V([1, 1, 1, 1], 1) > V([2, 2, 1, 1], 1)$, establishing that no worker would ever be willing to quit the sector 1 job during state 1.

Appendix B

Here I show that under assumptions (31)–(33) and $M > m$, no worker would choose a permanent commitment to sector 2. Note first that for any of equilibria A–C, if one is to be employed in sector 2, state 1 is the most favored, followed by 3, 2, and then 4:

$$\begin{aligned} &u_2(X(1), L_1(1), L_2(1)) \geq u_2(X(3), L_1(3), L_2(3)) \\ &\geq u_2(X(2), L_1(2), L_2(2)) \geq u_2(X(4), L_1(4), L_2(4)). \end{aligned}$$

The particular permanent sector 2 strategy that is optimal for the worker depends on where $\bar{u}$ would be placed in this chain of inequalities, but clearly the optimal choice must be one of (0, 0, 0, 0), (2, 0, 0, 0), (2, 0, 2, 0), (2, 2, 2, 0), or (2, 2, 2, 2).

Now (0, 0, 0, 0) is dominated by (1, 0, 0, 0) by assumption (32). Likewise, (33) implies that (2, 0, 0, 0) is dominated by (1, 0, 0, 0) and that (2, 0, 2, 0) is dominated by (1, 0, 1, 0). We saw in the text that (2, 2, 2, 2) is dominated by (1, 0, 0, 2), leaving (2, 2, 2, 0) as the sole element to be considered. But in the case B equilibrium, this is dominated by (2, 2, 2, 2), whereas in the case A equilibrium, it is dominated by one of (2, 2, 2, 2) or (2, 0, 2, 0), both of which were dismissed above.

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