

# In-Class Midterm 2

This examination is open-book, open-notes. Other people are closed.

Notation not defined here is taken from Starrs General Equilibrium Theory. State any additional assumptions you need.

## 1

Recall

**Intermediate Value Theorem** Let  $[a, b]$  be a closed interval in  $\mathbf{R}$  and  $f$  a continuous real-valued function on  $[a, b]$  so that  $f(a) < f(b)$ . Then for any real  $c$  so that  $f(a) < c < f(b)$  there is  $x \in [a, b]$  so that  $f(x) = c$ .

Starr's *General Equilibrium Theory*, problem 7.1. Consider a two-commodity economy with an excess demand function  $\tilde{Z}(p)$ .  $p \in P = \{p \mid p \in \mathbf{R}^2, p \geq 0, p_1 + p_2 = 1\}$ . Let  $(p)$  be continuous, bounded, and fulfill Walras' Law as an equality ( $p \cdot \tilde{Z}(p) = 0$ ), and assume  $\tilde{Z}_1(0, 1) > 0$ ,  $\tilde{Z}_2(1, 0) > 0$ . Use the intermediate value theorem to show that the economy has a competitive equilibrium.

## 2

Related to Starr's *General Equilibrium Theory*, problem 7.9. Consider a two-person, two-commodity pure exchange economy, an Edgeworth Box, with  $X^i \equiv R_+^2$  for both households. Assume axioms C.I - C.V, C.VII, C.VIII. This is an economy without active production so assume  $\mathcal{Y}^j \equiv \{0\}$  (the set of the zero vector) for all  $j$ . Note that this specification of  $\mathcal{Y}^j$  trivially fulfills P.II, P.III, P.V, P.VI.

Recall Theorem 7.1: Assume P.II, P.III, P.V, P.VI, C.I-C.V, C.VII, and C.VIII. There is  $p^* \in P$  so that  $p^*$  is an equilibrium.

Demonstrate that the Edgeworth Box has a competitive equilibrium price vector.

## 3

From (first edition) Starr's *General Equilibrium Theory*, problem 7.2. Consider the general competitive equilibrium of a production economy with redistributive tax-

ation of income from endowment. Half of each household's income from endowment (based on actual endowment, not net sales) is taxed away. The proceeds of the tax are then distributed equally to all households. We thus have

$$M^i(p) = p \cdot (.5r^i) + \sum_{j \in F} \alpha^{ij} p \cdot y^j + T,$$

where  $T$  is the transfer of tax revenues to the household,

$$T = (1/\#H) \sum_{h \in H} p \cdot (.5r^h).$$

Assume the household consumption sets are  $R_+^N$  and that household endowments are  $r^i \gg 0$  (endowments are strictly positive in all goods). Assume that the households treat  $T$  parametrically (they do not take account of the effect of their own consumption decisions on  $T$ ).

Assume P.I - P.VI, C.I-C.VIII. Note that the redistributive taxation is equivalent to considering an alternative version of the original economy with endowment's differing by a transfer prior to starting economic activity. Assume there exists a competitive equilibrium in the economy.

Is the resulting general equilibrium allocation Pareto efficient? That is, does the First Fundamental Theorem of Welfare Economics (Theorem 12.1) correctly apply? That theorem is proved in a setting without taxation. Can it correctly apply in this setting?