Chapter 1

THE TRANSACTIONS ROLE OF MONEY*

JOSEPH M. OSTROY

University of California, Los Angeles

ROSS M. STARR

University of California, San Diego

Contents

*We are indebted to Eduardo Siandra for research assistance.

Handbook of Monetary Economics, Volume I, Edited by B.M. Friedman and F.H. Hahn

1. Introduction

Of the three commonly acknowledged roles that money plays – unit of account, store of value and medium of exchange – it is in the last role as a facilitator of transactions, or essential lubricant to the mechanism of exchange, that money first comes to our attention.

The transactions role of money cannot be separated from its function as a store of value. If after the sale of one commodity for money, but before the purchase of another commodity with it, money perished, it could hardly serve as a medium separating purchase from sale. Though a medium of exchange must necessarily be a store of value, stores of value are not necessarily money. What distinguishes money from other stores of value is its liquidity, and what underlies the liquidity of money is the fact that it is the common medium through which other commodities are exchanged. We shall not define "liquidity" here (see Chapter 2 by Hahn in this Handbook); but the essential points are that liquidity is the ready convertibility through trade to other commodities and that it is a property not of the commodity itself but something that is established through the trading arrangement.

The transactions role of money can be readily separated from its usage as a unit of account by observing that the unit of account might be pounds of salt or a more stable foreign currency without being a medium of exchange. There are certainly good reasons why the medium of exchange should also serve as a unit of account, although this matter has not received much attention. Nevertheless, and despite the widespread observation that the unit of account is almost always the medium of exchange, there is widespread agreement that the unit of account is the least significant of money's three roles.

These commonplace observations would seem to confirm the conclusion that among money's three roles, the transactions one is at the top of the hierarchy. However, as measured by the attention paid to it in monetary theory, the transactions role is a distant second to the store of value. Perhaps this is as it should be. After all, some of the more important propositions in the history of monetary theory concern the "veil of money" or the "neutrality of money", phrases suggesting that although we could not do without the lubricating functions of a medium of exchange, they may be taken for granted to get onto more important matters. Simply because a property is unique does not imply that it is worthy of special attention. The fact that money is always the medium of exchange, just as it is almost always the unit of account, does not necessarily mean that in this it is economically more significant than in its role as a store of value, even though it shares that property with other durable goods.

But the dominance of the store of value over the medium of exchange function in monetary theory is not the conclusion of a openly contested debate. The game was rigged from the start in the sense that the prevailing theory, especially general equilibrium theory, could accommodate the store of value function more readily than it could the medium of exchange. In this chapter we shall report on recent developments to make general equilibrium theory a more hospitable setting for the transactions role of money. In the remainder of this introduction we shall cast a quick backward glance at the historical tradition, point out some critiques of this tradition that have helped to shake its grip, and then go on to summarize, with the aid of some informal stories, where we shall be heading.

A *caveat*: We shall confine ourselves to general equilibrium rather than partial equilibrium approaches to our subject. Thus, even though the phrase "transactions role of money" has in the past been virtually synonymous with the work of Baumol (1952) and Tobin (1956), that literature will not be discussed below.

1.1. The Walras–Hicks–Patinkin tradition: Integrating money into value theory

Walras (1900) not only gave us the first systematic account of general equilibrium theory, he was also conscientious in his efforts to incorporate money into it. Above all, he sought to incorporate money in a way that would be consistent with the rest of his scheme. Walras accomplished this by making a distinction between the stock of money, an object without any utility of its own, and the "services of availability" of the stock, which does enter into household utility functions and firm production functions. Thus, money is put in a similar footing with other (capital) goods and an equation of the offer and demand for money can be derived from the utility-maximizing hypothesis.

Walras' suggestion, coming as it did in an advanced theoretical treatise when marginal analysis was still a novelty, was ahead of its time. By the 1930s, Hicks (1935), who had certainly absorbed the lessons of Walras, could see the logic of Walras' approach. For Hicks, "marginal utility analysis is nothing other than a general theory of choice" and since money holdings can readily be regarded as choice variables, the obvious methodological conclusion was that monetary theory can and should be incorporated into a suitably generalized version of value theory.

Patinkin's treatise (1965) represents the culmination of this tradition. Here was a comprehensive statement of many of the key ideas in modern monetary theory and macoeconomics carefully constructed along value-theoretic lines.

The unstated presumption of the Walras–Hicks–Patinkin tradition was that without being firmly embedded in the more rigorous choice-theoretic general equilibrium principles of value theory, monetary theory would be weak and undisciplined. What this tradition did not question was the capacity of the existing value theory to accommodate the challenge of monetary exchange. The goal was the integration of monetary and value theory but it was understood that this would be achieved by integrating monetary theory *into* the structure of existing value theory.

1.2. Critiques of the tradition

The clarity and comprehensiveness of Patinkin's presentation helped to make it the standard general equilibrium account of monetary theory. Unavoidably, it became an object of closer scrutiny. Of the many commentaries on Patinkin's work, we point to two that were influential in stimulating a fresh look at the transactions role of money.

Hahn (1965) posed a basic existence problem: Does a model of a monetary economy have an equilibrium? In addition, what guarantees that all of the equilibria to such an economy are monetary rather than barter, i.e. ones in which the price of money is zero?

With the individual's demand for money arising from the presence of real money balances in the utility function, what happens when the individual's real money balances are zero? Real money balances may be zero for two very different reasons. First, the price of commodities in terms of money may be positive, but the individual's nominal balances are zero. Second, the individual's nominal balances may be positive, but the price of commodities in terms of money may be infinite. According to one reasonable scenario, in the second case where money is worthless there would be no demand for it. Hahn points out this leads to the conclusion that there exists a non-trivial, non-monetary equilibrium. It is only under the more dubious assumption that when money is worthless there is a positive demand for nominal balances, that Hahn is able to show the existence of an equilibrium with a postive price for money.

This is the lesson that Hahn drew from his thought experiment.

> All this suggests that while Patinkin has rendered signal services he has failed to provide a model which can serve as an adequate foundation for monetary theory. Such a model, it seems to me, must have two essential features beside price uncertainty. It must distinguish between abstract exchange opportunities at some notionally called prices and actual transactions opportunities.

In an exchange economy, putting money, even real money balances, into the utility function is an unreliable choice-theoretic short cut for modelling the transactions role of money. Suppose that the utility function is $u(x, m/P)$, where x is a vector of non-money commodities and m/P is nominal money balances divided by some index of the price of non-money commodities. These tastes are given independently of w the initial endowments of non-money commodities. At prices p for the non-money commodities, suppose that the utility-maximizing demands are $x(p)$ and $m(p)$. Now, change the initial endowment so that $w = x(p)$, i.e. at the prices p the individual does not want to trade. Note that the utility function and the marginal rate of substitution between real money balances and other commodities remains the same whether or not the individual plans to trade. In other words, the transactions role of money is not well approximated by simply putting money into the utility function.

Clower (1967) continued the attack. He focused directly on the description of the household budget constraint. In a money economy, "money buys goods, goods buy money, but goods do not buy goods". The last injunction about goods not directly buying goods is not implied by the standard general equilibrium budget constraint used by Patinkin. This constraint is simply the accounting identity that the total value of all purchases must equal the total value of all sales.

There are conditions under which the accounting constraint would be consistent with the monetary exchange injunction. If individuals supplied labor services to firms for which they received money and purchased commodities with that money from firms, then for every dollar of sales of labor services there would be a dollar with which to buy commodities. Feenstra (1986) uses such conditions to establish a kind of equivalence between the money-in-the-utility function and money for transactions purposes approaches. But even if such restrictions were imposed – certainly not in an exchange economy – it seems appropriate to allow the restriction to appear explicitly through the budget constraint rather than implicitly through the utility function.

Clower's critique led to a position similar to Hahn's:

> The natural point of departure for a theory of monetary phenomena is a precise distinction between money and nonmoney commodities. In this connection it is important to observe that such a distinction is possible only if we assign a special role to certain commodities as means of payment.

The lesson, according to Clower, is that exchange is a relation among commodities and monetary exchange is evidence that the relation is asymmetric. To capture this asymmetry, he proposed what has come to be known as the *cash-in-advance* constraint. Dividing commodities into those that are non-money and the money commodity, let $z = (z_i)$ be the vector of net trades of non-money commodities and let $z_m = M - M_0$ be the net trade in money, the difference between final holdings $M \geq 0$ and the initial money balances held at the start of the period M_0. To the standard budget constraint,

$$p \cdot z + z_m = 0\,,$$

Clower proposed the addition of

$$p \cdot [z]^+ \leq M_0$$

to capture the idea that purchases, $[z_i]^+ = \max\{z_i, 0\}$, must be financed by money on hand rather than by sales of other commodities as the standard budget constraint allows.

This proposal is not without its difficulties, and one might be tempted to say that it is as arbitrary as the earlier practice it was designed to replace of putting money in the utility function. One difficulty is that the proposal originates from a rather synthetic position – that exchange is a relation among commodities. A closer look at the rationale for a common medium of exchange reveals a more satisfying starting point: exchange is a relation among individuals. It is from the problematics of this relation among individuals that we can understand the function of an asymmetric relation among commodities involving a medium of exchange. Another difficulty is that as an added constraint, it cuts down on the exchange opportunities available with the standard budget constraint. Is this restriction gratuitous or is it symptomatic of the features of a money economy that does not operate according to the frictionless barter ideal? Whatever questions it raises, the proposal does provide an indisputable transactions role for money, something that is lacking in the Walras–Hicks–Patinkin tradition.

1.3. Some parables of monetary exchange

In this subsection we relate some simple stories of exchange relations among individuals. They are designed as introductions to the more formal models, described in Sections 2–5. Their purpose is to illustrate among a variety of possible transactions scenarios the common denominator for monetary exchange. That common denominator is the problem, taken for granted in traditional theory, of enforcing budget constraints.

1.3.1. A pair of Robinson Crusoes

Two elderly, largely self-sufficient gentlemen live on an island. Having only the most anemic impulses to truck and barter, their sole contact is the irregular exchange of dinners. Since both agree that meal preparation is onerous, they take turns. However, because dinners are exchanged so infrequently and because their memories are not what they used to be, these Robinson Crusoes cannot always agree on who gave the last dinner. On several occasions both have claimed to have provided the last meal. Each gentleman recognizes that this is a self-serving claim since this is what each would like to remember, but neither is sufficiently confident of his recollection to be sure of the truth. These disagreements have produced so much tension and ill-will that dinners are now exchanged even less frequently.

To attenuate this problem, the one who is coming to dinner next picks up a stone and paints it an artificially colored green to distinguish it from other stones and brings it to his host. At the next planning session for a dinner, the most recent host will be reminded by the presence of the green stone that it is his turn to be invited, and he will be expected to bring the stone with him when he arrives. Indeed, without receiving the stone the host may feel justified in turning away his guest as not having the required evidence of an invitation.

This quite rudimentary story reveals an essential feature of monetary exchange. Money is a commonly acknowledged record-keeping device. Here the only information about the past which has to be recorded is who gave the last dinner. Each gentleman "pays" for his dinner by transferring the record of this fact to the other.

1.3.2. Record-keeping at a central clearing-house

Let us separate into two parts the problem of equilibrium in exchange. First, there is the problem of finding market-clearing prices for which we invoke the mythical auctioneer. The second problem has to do with the actual execution of these trades. If the auctioneer had knowledge of every individual's excess demands and supplies, there is a centralized solution. The auctioneer could simply feed this information into a transportation-type computer algorithm and upon receipt of an answer instruct each individual to transfer specified quantities of commodities to certain other individuals. But this is more information than the auctioneer is typically presumed to have. In searching for equilibrium prices, it is only aggregate excess demand (or supply) for each commodity that is required for the auctioneer to find equilibrium prices, not its detailed distribution across individuals.

Consider the execution problem for a clearing-house with no inventories and only the information that aggregate excess demands are zero. We shall also impose the logistical limitation that individuals cannot trade with the clearing-house all at once. This will mean that at least the first few traders who come to drop off excess supplies will not be able to take away all of their excess demands. Again there is a record-keeping problem.

An obvious record-keeping device is for the clearing-house to issue blue chips to each person in the amount of the excess of the value of the goods supplied compared to those received. Because prices are fixed at market-clearing values, each person can silently spend his chips on the available supplies when he returns to the auctioneer, knowing that all supplies will eventually be claimed by those who have a demand for them.

1.3.3. Exchange without a clearing-house

After finding and announcing prices at which excess demand for each commodity is zero, suppose the auctioneer retires from the scene. It is up to the individuals themselves to fulfill their own excess demands. Trade takes place not at a central depot but in a decentralized setting in which opportunities for exchange are presented as a sequence of meetings between pairs of individuals. Suppose, further, that each person wishes to minimize the numbers of trades, i.e. periods, to fulfill his excess demands.

If each person knew the entire configuration of everyone's excess demands as well as the order of pairwise meetings, the minimization problem would again admit a somewhat more complicated but nevertheless similarly centralized solutions as in the previous auctioneer parable. Here the solution would require individuals to trade not only on their own account but to act as intermediaries, taking goods to be passed on to others. This minimum time and trade algorithm of barter exchange would often contradict the rule of quid pro quo – the stipulation that the value of each pairwise net trade evaluated at Walrasian market-clearing prices is zero. Even though each person aims to execute an overall net trade with zero market value, the most efficient way to accomplish this in a sequence of pairwise trades is not to constrain each bilateral commodity transfer to be zero.

When individual excess demands are not single-valued at equilibrium prices, there is a further problem. Markets may clear at certain prices for a specified vector in the aggregate excess demand correspondence; however, when individuals come to execute their trades they need not pick the ones consistent with market clearance. It is tempting to dismiss this as a minor qualification and certainly we shall not dwell on it. Nevertheless, it is symptomatic of an issue that cannot be dismissed: the amount of information that is compatible with equilibrium when the problem of executing trades is ignored – in this case, just prices – can be much too coarse to accommodate what has to be known to execute trades. See Townsend and Wallace (1987) and Benveniste (1987).

It may be objected that once an individual moves away from his Walrasian "budget-line" by making a non-zero-valued net trade, there is no reason why there should be an eventual return. For example, once a positive-valued net trade is made so that an individual's wealth has been augmented, why should he then agree to a wealth-reducing trade later on? While this doubt will be recognized below, it need not apply here because we are operating under the hypothesis of complete information – everyone knows all excess demands. In this setting undetected cheating is impossible. An individual who takes more than he gives at some pairwise meeting is simply executing a part of the overall plan to which the members of the economy have submitted themselves. It is as though the participants are agents in a firm carrying out their assigned tasks in front of each other. The lesson we draw is that in a world of complete information the requirements for enforcing overall budget balance are met, so quid pro quo is an avoidable constraint on the transactions process.

The thought experiment demonstrates by contradiction the importance of *incomplete information* as a key determinant of "the inconveniences of barter" underlying monetary exchange. Assume to the contrary that individuals know only their own excess demands. Even if one knows the order of the names of one's future trading partners, one does not know very much about them. This has two important consequences. First, there are the hazards of indirect trade. It is no longer possible to know that you can take goods to pass onto others. Second, the quid pro quo constraint comes into its own. Technologically, there may still be advantages to violating the quid pro quo even if individuals eschew indirect barter and trade only on their own account. But there is an incentive to cheat because it is impossible to distinguish between a person who is honestly trying to fulfill his excess demands and someone who is only pretending.

Trader A: I want a candybar. I just gave two bottles of wine to someone and I deserve a candybar as part of my compensation.

Trader B: Why did you give up two bottles of wine?

Trader A: The person I gave it to said he had previously given up a bushel of wheat.

How does monetary exchange resolve this problem? Not by removing any technological transaction costs, but by changing the relative rewards associated with various strategies. The problem is how to enforce the overall budget constraint underlying market-clearing excess demands while also permitting individuals temporarily to violate these constraints in the course of fulfilling those excess demands. Again, what is called for is a record-keeping device. One commodity with utility like any other could serve that function as a kind of physical record provided each person had sufficiently plentiful supplies of it. The willingness of an individual to part with it in payment for an excess of the value of commodities received over those given up is evidence that he is willing to bear the cost of his purchase. Of course a commodity record-keeping device is a relatively crude instrument. The same function could be abstracted by a system of electronic fund transfers provided that accounts were monitored by an agency with sufficient police powers to punish "overissuers".

The previous examples have introduced money as a device for implementing exchange without entering essentially into the determination of the equilibrium allocation. The trades predicted by the frictionless barter theory could then be implemented once money is introduced to deal with the frictions. If that were always the case, money would indeed be a veil. Such a conclusion is not warranted: monetary equilibrium can be quite distinct from barter with or without frictions. This is the purpose of the next example.

1.3.4. A chicken–egg economy

Each of a large number of individuals has his own chicken which has a 50—50 chance of laying an egg each day. The chickens have no value when sold because they lay eggs only for their original owners. There may be trade in eggs but not in chickens.

Ideally, the large numbers of individuals should permit the members of this economy to take advantage of the law of large numbers, namely each would exchange the 50–50 probability of an egg each day for the virtual certainty of one-half of an egg (eggs are divisible). Assuming risk aversion on the part of the owners, this would be an efficient allocation. But here again we must deal with the consequences of the technological transactions costs of incomplete information, especially private information. The chicken's owner is the only one who knows whether it has laid an egg. Thus, the proposed ideal trade to take advantage of the law of large number is unenforceable. What is to prevent someone whose chicken has had an average run of eggs from pretending that it is below average, or someone with an above average run pretending that it is only average, and pocketing the difference between actual and stated egg production.

It is clear that for trades to be enforceable they must be dependent on one's actual trading history and not on ex ante expected values while for trades to be efficient the reverse must be true. In this parable, there is no record-keeping/monitoring device that will resolve the problem of executing efficient trades.

What kind of trading arrangements are possible? Consider the following alternative. When you buy an egg you write a check in favor of the seller who deposits it to the credit of his account and which is automatically debited from the buyer's account so cumulative records of each person's trading history are kept by an outside authority (not by the person himself). So far, this is just information-gathering. The teeth of such a device is contained in the restriction that one cannot have a cumulative balance below a certain number of (negative) eggs. Once that level is reached, consumption must be curtailed until one's balance rises above this level. Now there is no such thing as cheating. One may consume at any rate provided this does not conflict with one's minimum egg balance. This means that one is forced to perceive the costs of additional consumption today in terms of the increased probability of having to curtail future consumption.

1.4. Introducing general equilibrium theory to monetary exchange

In each of the stories discussed above we found that the transactions role of money calls into question the modelling of trade via a single budget constraint. Repeatedly, we found on closer examination of the problem of implementing trades, the single budget constraint must be replaced by a sequence of budget constraints. This was the role of the green stone in the story of the two Robinson Crusoes. The exchange of dinner for the stone brought the accounts into balance period by period so that the sometime dinner companions did not have to rely on their memories. The introduction of blue chips in the story of the central clearing-house or the parable about the auctioneer who retired from the scene after equilibrium prices were announced led to a similar conclusion – the need for a tracking device to monitor departures from the overall balance during the interim trading periods so as to ensure that individuals would stick to a trading plan the overall net value of which is zero. In the chicken–egg economy, the issue is once again that a single idealized intertemporal budget constraint is unworkable and is replaced by what is effectively a sequence of overdraft constraints.

The stories vary in the extent of which the responsibility for trade is left to an outside authority and is therefore centralized or is left to the individuals themselves and is therefore more decentralized. The greatest contrast is between the clearing-house and the story about the auctioneer who retires from the scene to leave the individuals to trade on their own in pairs. The latter is the only one in which the purely logistical problems of exchange are present. Even in the chicken–egg economy, there is no difficulty in bringing buyers and sellers together to exchange.

Despite the variation in the levels of *logistical* decentralization in these stories, there is a common element of *informational* decentralization, and it is this element that is directly responsible for the change from a single budget constraint to the imposition of a sequence of budget constraints. The common element is the private information each individual has about his own circumstances. Even in the most logistically centralized story, the clearing-house does not know individual excess demands.

Informational decentralization does not by itself lead to the imposition of the sequence of budget constraints but the causal connection is rather immediate. It is the presence of private information that leads to an obvious *moral hazard* problem when individuals face only a single intertemporal budget constraint in the course of executing intertemporal trades. The lack of enforceability of this single constraint leads to the necessity of a temporal sequence of constraints.

In the following sections we describe some of the contributions that have aimed at elaborating the transactions role of money in a theory of exchange. We begin in Section 2 with the program of modern general equilibrium theory to meet the challenge of finding an internally consistent role for money. In Section 3 we consider models that do not precisely fit the modern general equilibrium mold but ones that focus on the logistics of bilateral exchange. These models serve as a vehicle for displaying the disadvantages of barter commonly believed to be fertile ground for uncovering the transactions role of money. Also, in Section 3 we consider the related problem of how one or more commodities might emerge as media of exchange. In Section 4 we look more closely at the implications for the allocation of resources when the budget enforcement problem cannot be taken for granted. In Section 5 we focus on the development and application of one model of a sequence of binding monetary constraints, the *cash-in-advance* constraint proposed by Clower. Concluding comments are contained in Section 6, and Section 7 provides a brief guide to some related work not specifically referenced in the text.

2. The modern general equilibrium transactions costs approach

At about the same time that Patinkin's *Money, Interest and Prices* appeared, advances were being made in general equilibrium theory. Of particular importance for our purposes were the conscious efforts to include time-dated and event-contingent commodities. This work, associated with the names of Arrow and Debreu, culminated in the publication Debreu's *Theory of Value* (1959). In this section we report on efforts to understand the transactions role of money via models directly inspired by these post-Walrasian contributions. This meant that the problem was viewed from the perspective of "an Arrow–Debreu economy", i.e. the inability to accommodate monetary exchange was attributed to the presence of certain theoretical devices used in the model and described below. Out of this frictionless framework, the response was to model frictions as the transactions costs of making certain kinds of exchanges, from which a role for money could be deduced.

We describe first the frictionless non-monetary economy. Goods are defined by their characteristics, location, and date of delivery. This formal structure corresponds to a fully articulated system of futures markets.[1] A household's

[1]Uncertainty can be treated in the model by augmenting the description of a commodity to include the state of the world in which it is deliverable. A contract is then characterized by what it promises, when, where, and under what realization of uncertain events. This formal model of the full set of Arrow–Debreu contingent contracts requires active markets in a variety of distinct instruments, many more than are generally available and actively traded in actual economies. A partial reduction in the large number of contracts needed can be achieved by the use of Arrow securities (Arrow insurance contracts) [Arrow (1964)], which specify a credit to the holder in numeraire on realization of a specified event.

endowment consists of goods (presumably including labor) available at a variety of dates. Its budget is simply the value of this endowment plus the present discounted value of the streams of future profits of the share of business it owns. This budget is in the nature of a lifetime budget constraint. The presence of the futures markets eliminates the distinction between income and wealth. Given this lifetime budget constraint, the household allocates its wealth to the purchase of present and future consumption. It acquires a portfolio of present goods and of contracts for future delivery of its desired consumption plan sufficient to exhaust its budget constraint. Firms buy contracts for present and future inputs, and sell contracts for present and future output to maximize the present discounted value of profits.

Once these contracts are fully arranged, the balance of economic activity consists of their fulfillment. The intertemporal allocation process in the Arrow–Debreu model can be described in the following way: there is a single date of active trade where trade takes place in current goods and in futures contracts for future delivery of goods. There is no need for markets to reopen in the future – all desirable trades have already been arranged. But absent reopening of markets, there is no function for money. Hence, the Arrow–Debreu theory establishes sufficient conditions for money to be useless and positively denies it any intertemporal allocative function.

Though striking, this conclusion is unsurprising; money has no job to perform here because its job is being done by futures markets. The Arrow–Debreu futures markets are designed to perform all intertemporal allocative functions. These are essentially two: price and output determination at each point in time and intertemporal reallocation of purchasing power. The first function can of course be performed by spot markets (with intertemporal perfect foresight). The second is a capital market function. The futures markets allow sales of current output to finance future acquisitions, and they allow sales of future output to finance current purchases. The result is that money and debt instruments, devices for the intertemporal transfer of purchasing power, are otiose. The equilibrium allocation is Pareto efficient. Any technically possible reallocation giving an intertemporal consumption plan preferable for some households would necessarily degrade the well-being of others.

Conversely, if the conditions that allow the futures markets so fully to exercise their function are absent, we may expect a role for money and for futures markets in money (debt instruments). In fact, futures markets for most goods are generally inactive (loosely speaking, do not exist). The reason for this is the structure of transactions costs, which favors spot over futures transactions. In the presence of differential transactions costs on spot and future transactions, the Arrow–Debreu model becomes inapplicable and we are led to a sequence economy model [Hahn (1971)].

The basic idea of a sequence economy is that markets reopen over time.

There is a single essential revision of the Arrow–Debreu model that creates a role for money: require that budget balance be fulfilled at each trading date rather than only in the lifetime budget constraint. In the sequence economy, budget constraints apply at each date. At each date, the value of goods and contracts for future delivery that an agent sells to the market must be at least as large as the value of goods and contracts he accepts from the market. The rationale for this constraint is strategic: this is how the lifetime budget constraint is reliably enforced in an intertemporal setting.

Differential transaction costs on spot and futures markets (in particular, higher costs on futures) imply that markets may reopen over time and agents will face a budget constraint at each date. For a given agent, the time pattern of the value of his planned consumption may differ from that of his endowment. In order to implement such plans consistent with the sequence of budget constraints, futures markets may be used or real assets held intertemporally as stores of value. Transaction costs are not a source of inefficiency in themselves. Part of economic activity is the reallocation of goods and resources. If the reallocation process is a resource-using activity, then resources devoted to it are engaged productively. Any durable good, ownership claim, or futures contract can perform the function of shifting purchasing power forward or back in time, but transaction and storage costs associated with some instruments may be wasteful. These are real resource costs incurred only for the fulfillment of sequential budget constraints and technically unnecessary to implement the allocation. These costs, and any reallocation of consumption plans undertaken to avoid them, represent an efficiency loss. Introduction of fiat money – assumed to be transaction costless – results in an allocation that is closer to Pareto efficient. Households may transfer purchasing power over time by accumulating and depleting their money balances and by the use of money futures contracts (loans). If the desired timing of purchase and of sale transactions do not coincide, money acts as a carrier of purchasing power between the two transaction dates. Money restores allocative efficiency by allowing both fulfillment of the sequential budget constraints and the use of only spot goods transactions, without distortion of the lifetime consumption plan.

Transaction costs, like prices, are correctly computed as present discounted values. Hence, one reason that transaction costs may be higher on futures markets than on spot markets is the timing with which the costs are actually incurred. Costs are incurred at the transaction date and at the delivery date. The present discounted value of the transaction cost for a spot transaction planned now for execution in the distant future will be small. In contrast, the transaction cost on a futures transaction conducted in the present, but with the same distant delivery date, may be substantial. Hence, time discounting notably strengthens the argument for reliance on spot rather than futures markets. Under uncertainty, the argument for spot markets as a low-cost device is again strengthened by considering Arrow–Debreu contingent commodity futures or Arrow securities markets as the alternatives. Because of the multiplicity of contingencies, many contingent futures contracts will be written that will not be executed by delivery. Furthermore, contracts contingent on the occurrence of specified events are necessarily costly to write and enforce. Subject to a different, possibly Pareto inefficient, allocation of risk-bearing, a reduction in the number and complexity of transactions and a corresponding reduction in transaction costs is achieved by reliance on spot rather than futures markets.

2.1. Pareto inefficient equilibrium in a non-monetary economy: An example

Let there be two households, A and B. There is one good available at two dates: 1 and 2. Both households have the same utility function,

$$u(c(1), c(2)) = [c(1)]^{1/2}[c(2)]^{1/2} ,$$

and the same transaction technology,

$$z_1(1) = \tfrac{1}{2}x_2(1) + \tfrac{1}{2}y_2(1) ,$$

$$z_2(2) = 0 ,$$

where $c(t)$, $t = 1, 2$, is the consumption at date t; $z_\tau(t)$ is the transaction cost incurred at date τ from transactions conducted on the market at date t, $x_\tau(t)$ and $y_\tau(t)$ are the purchases and sales, respectively, on the market at t for delivery at τ. That is, spot transactions are costless in both periods, futures purchases and sales are costly. The households differ in endowment:

$$\omega^{A}(1) = 2 , \qquad \omega^{A}(2) = 0 ,$$

$$\omega^{B}(1) = 0 , \qquad \omega^{B}(2) = 2 ,$$

where $\omega^i(t)$ is i's endowment of spot goods at t.

Plainly, since spot transactions are costless and futures are costly, an efficient allocation will use spot markets only. But in order to fulfill the budget constraint in each period, A must buy any period 2 consumption he requires on the futures market thereby incurring the cost in current goods $\frac{1}{2} \cdot x_2^{A}(1)$ per unit for his futures purchase. Similarly, B must sell future endowment on the period 1 market to finance current consumption and current transaction costs. The transaction costs put a wedge between buying and selling (shadow) prices:

$$\mathrm{MRS}^{\mathrm{A}}_{1,2} = \frac{p_1(1)}{p_2(1) + (1/2)p_1(1)}\,,$$

$$\mathrm{MRS}^{\mathrm{B}}_{1,2} = \frac{p_1(1) + (1/2)p_1(1)}{p_2(1)}\,,$$

where $p_\tau(t)$ is the price on the market at date t for the good delivered at τ.

The difference in MRSs facing A and B is indicative but not conclusive of an allocative inefficiency. If the transaction costs faced by A and B in trade were necessary to achieve a reallocation, then the spread in their MRSs would represent an unfortunate necessity. But given the transaction technology available, such costs are not inevitable. Spot transactions have no costs. The spread in MRSs comes from the use of futures markets with their attendant higher transaction cost. Any allocation that uses futures markets here is necessarily inefficient. Only the sequential budget constraint mandates the use of futures markets and represents the source of Pareto inefficiency. Efficient allocations in the example are characterized by the use of spot trade only and $\mathrm{MRS}^{\mathrm{A}}_{1,2} = \mathrm{MRS}^{\mathrm{B}}_{1,2} = 1$.

2.2. *Intertemporal transactions cost models: Sequence economy*

We now present a formal pure exchange sequence economy model with transactions costs [Hahn (1973), Kurz (1974a, b), Heller and Starr (1976)]. First the non-monetary version will be presented, then it will be extended to include money.

We are interested in four principal results here:

(I) Under suitable sufficient conditions involving continuity, convexity, and non-emptiness of demand functions, there exist market-clearing prices and an equilibrium allocation in the non-monetary economy.

(II) Contrary to the First Fundamental Theorem of Welfare Economics, in the model with transactions costs the allocation associated with the general equilibrium in (I) is not generally Pareto efficient. This is the result of applying the time sequence of budget constraints as opposed to the lifetime budget constraint of an Arrow–Debreu model.

(III) In the monetary economy additional assumptions assure non-triviality of the monetary equilibrium (a finite determinate monetary price level).

(IV) Even the non-trivial monetary equilibrium allocation of (III) is not quite sufficient to assure Pareto efficiency. The allocation will be Pareto efficient if there is perfect capital market. That is, the allocation is Pareto efficient if the transaction costs of money are nil and either a non-negative money holding condition is not a binding constraint or the transaction costs of notes (money futures) are also nil.

Commodity i for delivery at date τ may be bought spot at date τ or futures at any date t, $1 \leq t < \tau$. The complete system of spot and futures markets is available at each date (although some markets may be inactive). The time horizon is date K; each of H households participates in the market at time 1 and cares nothing about consumption after K. There are n commodities deliverable at each date. At each date and for each commodity, the household has available the current spot market, and futures markets for deliveries at all future dates. Spot and futures markets will also be available at dates in the future and prices on the markets taking place in the future are currently known. Thus, in making his purchase and sale decisions, the household considers without price uncertainty whether to transact on current markets or to postpone transactions to markets available at future dates. There is a sequence of budget constraints, one for the market at each date. That is, for every date the household faces a budget constraint on the spot and futures transactions taking place at that date, equation (4) below. The value of its sales to the market at each date (including delivery of money) must balance its purchases at that date.

In addition to a budget constraint the agent's actions are restricted by a transactions technology. This technology specifies for each complex of purchases and sales at date t, what resources will be consumed by the process of transaction. It is because transaction costs may differ between spot and futures markets for the same good that we consider the reopening of markets allowed by the sequence economy model. Specific provision for transactions cost is introduced to allow an endogenous determination of the activity or inactivity of markets. In the special case where all transaction costs are nil, the model is unnecessarily complex; there is no need for the reopening of markets, and the equilibrium allocations are identical to those of the Arrow–Debreu model. Conversely, in the case where some futures markets are prohibitively costly to operate and others are costless, then there is an incomplete array of active spot and futures markets.

All of the n-dimensional vectors below are restricted to be non-negative:

$x^h_\tau(t)$ = vector of purchases for any purpose at date t by household h for delivery at date τ.

$y^h_\tau(t)$ = vector of sales analogously defined.

$z^h_\tau(t)$ = vector of inputs necessary to transactions undertaken at time t; the index τ again refers to date at which these inputs are actually delivered.

$\omega^h(t)$ = vector of endowments at t for household h.

$s^h(t)$ = vector of goods coming out of storage at date t.

$r^h(t)$ = vector of goods put into storage at date t.
$p_\tau(t)$ = prices vector on market at date t for goods deliverable at date τ.

With this notation, $p_{it}(t)$ is the spot price of good i at date t, and $p_{i\tau}(t)$ for $\tau > t$ is the futures price (for delivery at τ) of good i at date t.

The (non-negative) consumption vector for household h is:

$$c^h(t) = \omega^h(t) + \sum_{j=1}^{t} (x_t^h(j) - z_t^h(j)) + s^h(t) - r^h(t) \geq 0 \quad (t = 1, \ldots, K). \tag{1}$$

That is, consumption at date t is the sum of endowments plus all purchases past and present with delivery date t minus all sales for delivery at t minus transactions inputs with date t (including those previously committed) plus what comes out of storage at t minus what goes into storage. We suppose that households care only about consumption and not about which market consumption comes from. Thus, households maximize $U^h(c^h)$, where c^h is a vector of the $c^h(t)$'s, subject to constraint.

The household is constrained by its transactions technology, T^h, which specifies, for example, how much leisure time and shoe-leather must be used to carry out any transaction. Let $x^h(t)$ denote the vector of $x_\tau^h(t)$'s [and similarly for $y^h(t)$ and $z^h(t)$].

A household's plan should be consistent with transaction technology,

$$(x^h(t), y^h(t), z^h(t)) \in T^h(t) \quad (t = 1, \ldots, K). \tag{2}$$

Naturally, storage input and output vectors must be feasible, so

$$(r^h(t), s^h(t+1)) \in S^h(t) \quad (t = 1, \ldots, K-1). \tag{3}$$

The budget constraints for household h are then:

$$p(t) \cdot x^h(t) \leq p(t) \cdot y^h(t) \quad (t = 1, \ldots, K). \tag{4}$$

Households may transfer purchasing power forward in time by using futures markets and by storage of goods that will be valuable in the future. Purchasing power may be carried backward by using futures markets.

Let $a^h(t) \equiv (x^h(t), y^h(t), z^h(t), r^h(t), s^h(t))$, the vector of h's actions on the market at date t. Let a^h be a vector of the $a^h(t)$'s h's K-period trading plan. Define x^h, y^h, z^h, r^h and s^h similarly. Define $B^h(p)$ as the set of a^h's which satisfy constraints (1)–(4). The household maximizes $U^h(c^h)$ over $B^h(p)$. $U^h(\cdot)$ is assumed to be continuous, concave, and monotone. Denote the demand correspondence (i.e. the set of maximizing a^h's) by $\gamma^h(p)$.

The correspondences $\gamma^h(p)$ are always homogeneous of degree zero in $p(t)$, as is seen from the definition of $B^h(p)$. We can therefore restrict the price space to the simplex. Let S^t denote the unit simplex of dimensionality, $n(K - t + 1)$. Let $P = \times_{t=1}^{K} S^t$.

An *equilibrium* of the economy is a price vector $p^* \in P$ and an allocation a^{*h}, for each h, so that $a^{*h} \in \gamma^h(p^*)$ for all h and

$$\sum_{h=1}^{H} x^{*h} \leq \sum_{h=1}^{H} y^{*h}$$

(the inequality holds coordinate-wise), where for any good i, t, τ such that the strict inequality above holds, it follows that $p^*_{it}(\tau) = 0$.

To ensure the existence of equilibrium, in addition to the usual conditions on preferences and endowments (convexity, continuity, positivity of income, etc.) we will need some structure on the transactions and storage technologies. These are just the sort of assumptions the general equilibrium model ordinarily requires of production technologies. The transactions and storage technologies should be closed convex sets including the origin, admitting free disposal, positive income net of transactions costs, and implying bounded levels of purchase and sale (no wash sales).

We have then

Theorem (Existence of equilibrium for the closed convex non-monetary economy). *In the non-monetary economy, under assumptions assuring non-emptiness, closure and boundedness of budget sets, there is a price vector $p^* \in P$ and an allocation $\langle a^{*h} \rangle_h$ such that*

$$a^{*h} \in \gamma^h(p^*),$$

$$\sum_{h=1}^{H} x^{*h} \leq \sum_{h=1}^{H} y^{*h},$$

with $p^*_{it}(\tau) = 0$ for i, t, τ such that the strict inequality holds.

If the sequence economy model were fully analogous to the Arrow–Debreu model we could then demonstrate that the allocation resulting from the equilibrium was Pareto efficient. The analogy is incomplete, however, and the result fails. Equilibrium allocations in the model may not be Pareto efficient. Money and a monetary equilibrium are required to overcome inefficiencies arising from the sequential structure of budget constraints (4). The illustration of this point is the previous example.

2.3. A monetary economy

The demand for goods in the sequence economy model is overdetermined: goods are desired as objects of consumption and as carriers of value between trading opportunities. The second demand may interfere with the first. When it does so, the introduction of a fiduciary or fiat money with negligible transactions and storage costs can change the equilibrium allocation to one that is Pareto efficient. It is important here that the private and social opportunity cost of holding money in inventory be negligible. If not, then an unnecessary wedge remains between buyers' and sellers' intertemporal MRSs, and first-order conditions for Pareto efficiency will fail. Since the opportunity cost of holding real goods in inventory will generally be non-negligible, there is an efficiency gain through the use of fiduciary (bank) or fiat money in place of commodity money.

The model may now be trivially modified to incorporate fiat money and bonds by introducing money as a zeroth commodity for which the household has no direct utility. The non-trivial modification is to ensure the existence of equilibrium with a positive price of money in each period. A futures contract for delivery of money is a bond (discounted note). Let $x^h_{0t}(t)$ [the zeroth component of $x^h_t(t)$] denote the total amount of spot money acquired by household h in the market at date t. Similarly, let $y^h_{0t}(t)$ be the disbursement of spot money at t. Now if $\tau > t$, then $y^h_{0\tau}(t)$ is a commitment made at time t to deliver $y^h_{0\tau}(t)$ units of money at date τ. Suppose that by convention each bond (discounted note) promises one unit of money. Then $y^h_{0\tau}(t)$ is the number of bonds with maturity date τ sold by h. Similarly, $x^h_{0\tau}(t)$ is the number of bonds purchased by household h with maturity date τ.

Spot money trades at a price of $p_{0\tau}(t)$ at date t [and $p_{0\tau}(t) = 0$ is thereby a possibility]. The equilibrium of a monetary economy is said to be *non-trivial* (that is, the economy is really monetary) if $p^*_{0t}(t) \neq 0$ for all t. The price of a bond maturing at τ is denoted $p^*_{0\tau}(t)$. With this convention we may write the budget constraints on the household as in (4), except that $p(t)$, $x(t)$, and $y(t)$ all contain an added group of zeroth components, one for each delivery date.

The interpretation is that spot money on the market at t can be acquired by spot and futures sales of goods and bond sales. Similarly, spot and futures purchases of goods and bond purchases can be paid for in cash, goods, bonds or goods futures.

What is to prevent a household from disbursing an unlimited amount of money $y^h_{0t}(t)$? The answer is to be found in constraint (1) for the zeroth component: "consumption" of money is a non-negative number. It is possible to disburse money for considerably more than one's current money holding without violating (1), but there must be corresponding receipts of money to balance out the discrepancy.

The volume of monetary trade planned in period t is simply the gross volume of planned spot money disbursements, $\sum_h y^h_{0t}(t)$, or receipts, $\sum_h x^h_{0t}(t)$. These will be equal in equilibrium. The demand for money (as a stock) at p is $\sum_h r^h_{0t}(t)$ where $r^h_{0t}(t)$ is an element of $a^h \in \gamma^h(p)$.

Money is held in this model as a low-cost means of intertemporal transfer of purchasing power. If bonds have a positive nominal interest yield, then non-interest-bearing money will be held to avoid the transaction costs of buying and liquidating bonds. In the convex cost case treated here money will then be held only for short periods where a bond's interest yield (over the short interval) would not compensate for the transaction costs of purchase and sale. In a non-convex transaction cost model [Heller and Starr (1976)], there is a tendency to concentrate transactions so that large intertemporal wealth transfers take place through bonds while small ones use money.

Since we are dealing with fiat (rather than commodity) money, utility maximization will always imply zero consumption of money. We shall assume that there is a positive endowment of spot money, at least at the beginning, and that at no time is there any input of cash being used up by the transaction process. Therefore, by (1),

$$r^h_0(t) - s^h_0(t) = \omega^h_0(t) + \sum_{j=1}^{t} [x^h_{0t}(j) - y^h_{0t}(j)] .$$

Thus, net additions to storage of cash at t equals endowment plus total net acquisitions from the market, where the total is taken over all previous transaction dates. Hence, a household needs to deliver cash only to the extent that his promises to others exceed the promises of others to him.

Transactions and storage technologies will also include money and bonds, since both of these may be costly to exchange. There is a further constraint that the household must satisfy: the terminal condition that holdings of money at the end of time should be at least equal to the money endowment. Without this artificial requirement, no one would want to hold a positive money stock at the end. This would drive money's terminal price to zero. But then no one would hold money at $K-1$, and so forth. The problem arises because of the use of a finite horizon. This restriction is characterized as

$$\sum_{\tau=1}^{K} [x^h_{0K}(\tau) - y^h_{0K}(\tau)] + s^h_0(K) \leq \sum_{\tau=1}^{K} \omega^h_0(\tau) \equiv M^h . \tag{5}$$

The constraint (5) says that the household is required to have at the economy's terminal date holdings of nominal money equal to its endowment thereof. One interpretation of this is that the money's issuer lends money at periods up to K and calls back its loan at the end of K.

As before, $\gamma^h(p)$ is the demand correspondence of household h, constrained to fulfill (1)–(5) (with the additional zeroth components corresponding to money and bonds). The correspondences $\gamma_t^h(p)$ are again homogeneous to degree zero in $p(t)$. Let S^t denote t the unit simplex of dimensionality $(n+1)(K-t+1)$. Let S_α^K denote the unit simplex of dimensionality $n+1$ with the restriction that $p(T) \in S_\alpha^K$ implies $p_{0K}(K) = \alpha$, where $0 < \alpha < 1$. Define

$$P^\alpha = \{(p(1), p(2), \ldots, p(K)) \mid p(t) \in S^t, p(K) \in S_\alpha^K\} .$$

By the same reasoning as before, γ_p^h is convex, compact and upper semicontinuous. Use of P^α as the price space amounts to assuming that the terminal price of money is exogenously set at $\alpha > 0$. A more elaborate economic rationale could be based on expectations [Grandmont (1977)] or terminal period taxation [Starr (1974)]. The use of infinite horizon models is more complex. Anchoring the terminal value of money at $\alpha > 0$ and assuming sufficient substitutability in consumption and transactions between periods results in a downward-sloping demand condition on money – if its value in preterminal periods goes down, nominal demand goes up.

We can now establish the existence of an equilibrium with a positive price of money for the monetary economy.

Theorem (Existence of non-trivial equilibrium for the monetary economy). *Under assumptions assuring non-emptiness, closure, and boundedness of budget sets, and assuring downward-sloping demand for money for any* α, $0 < \alpha < 1$, *there is a price vector* $p^* \in P^\alpha$ *and an allocation* $\langle a^{*h} \rangle_h$ *such that*

$$a^{*h} \in \gamma^h(p^*) ,$$

$$\sum_{h=1}^{H} x^{*h} \le \sum_{h=1}^{H} y^{*h} ,$$

with $p_{it}^*(\tau) = 0$ *for* i, t, τ *such that the strict inequality holds, and* $p_{0t}^*(t) \neq 0$ *for all* t.

The conditions so far developed are not quite sufficient to guarantee the Pareto efficiency of the allocation arising in a non-trivial monetary equilibrium. The reason is that (1), the non-negativity requirement applied to spot money, may be a binding constraint on some households in equilibrium. This can arise since there may be non-zero transaction costs in the money futures (bond) market generating a capital market imperfection. This difficulty need not necessarily arise. It will not if money holdings, r_{0t}^h, of all agents h at each date t, happen to be strictly positive in equilibrium. An alternative sufficient condition is that transaction costs on the money futures (bond) market be nil.

The source of Pareto inefficiency in equilibrium is a capital market imperfection deriving from the multiplicity of sequential budget constraints (4). When the cost of intertemporal transfer of purchasing power is not a binding constraint the resulting allocation is Pareto efficient. Intertemporal allocative efficiency will be achieved in three principal circumstances: (i) by simple good luck in the timing of endowments so that there is no need to use the capital market (an "inessential" sequence economy); (ii) when spot money stocks are strictly positive in equilibrium at all dates for all households; and (iii) when the transaction costs for spot and futures money are nil. Under any of these conditions, in equilibrium with a positive price of money in each period, the household demand problem subject to (4), the sequential budget constraint, is equivalent to the problem with a lifetime budget constraint. Then the First Fundamental Theorem of Welfare Economics from the Arrow–Debreu model applies. Hence, we are led to:

Theorem. *For any* $0 < \alpha < 1$, *let* $p^* \in P^{\alpha}$ *with* $p^*_{0t}(t) > 0$ *for* $t = 1, \ldots, K$, *be a non-trivial equilibrium price vector for the monetary economy fulfilling with equilibrium allocation* $\langle a^{*h} \rangle_h$ *and consumption plan* $\langle c^{*h} \rangle_h$. *Let money futures be transaction costless, or let* $r^h_{0t}(t) > 0$ *for all* h, *all* $t = 1, \ldots, K$. *Then* $\langle a^{*h} \rangle_h$ *is a Pareto-efficient allocation.*

The proof of the theorem consists of demonstrating that for each household, under the nil transaction cost conditions posited, the sequence of budget constraints (4) collapses to a simple lifetime budget constraint. Then the First Fundamental Theorem of Welfare Economics (a Walrasian equilibrium allocation is Pareto efficient) applies, and the theorem is proved.

The role of money in the sequence economy model is to provide the means of achieving an allocation consistent with a single lifetime budget constraint. The inefficiencies that occur in the non-monetary version of the model are those of an imperfect capital market: discrepancies across agents in their intertemporal marginal rates of substitution of consumption. Conversely, in an equilibrium of the fully monetized model, intertemporal MRSs are equated across all households.

The institutional distortion that plagues sequence economy models is the sequence of budget constraints – one for each period – facing agents as they trade. In order to pay for purchases, agents who wish to buy goods in one period must sell goods in the same period and vice versa. The resolution of the inefficiency is to sever the temporal link between commodity buying and selling transactions while continuing to fulfill the sequential budget constraint. The introduction of "money" is designed to achieve this. Money is defined as a commodity of positive price and zero transaction cost that does not directly enter in production or consumption. Rather than engage in costly futures trades to achieve budget balance at each trading date, traders use trade in money to bridge the gap in timing between desired sales and purchases. The assumption of zero money transaction cost is of course extreme but it captures the essential point: a major cost reduction relative to commodity trade. Money as a store of value with nil transaction costs spot and futures (a perfect capital market) allows undistorted intertemporal reallocation despite the sequential budget constraint.

3. The logistics of decentralized barter exchange

The starting point for this section is an issue that Walras had (wisely, for his purposes) glided over in his theory of multilateral exchange: the problem of exchange without a clearing-house described in Subsection 1.3.3. This problem is suggested by the idea that money eliminates the oft-quoted "disadvantages of barter"; and the efforts here are devoted to modelling the disadvantages of barter as a way of confirming and perhaps enlarging our intuition about the transactions role of money.

Despite the different starting points and matters of detail and emphasis, the conclusions reached both in this section and the previous one can be summarized in largely similar terms. The reasons differ, but each approach describes the overall gains from trade as occurring via a sequence of exchanges; and both point to the need for trades to be "balanced" (satisfy the quid pro quo) at each exchange, a need that conflicts with the potential for exploiting all the gains from trade. The role of money is to attenuate this conflict.

3.1. Dwelling on the disadvantages of barter

Jevons (1893) focused on the problems of coordinating trade among agents as the essential rationale for the use of a medium of exchange. He argued that without a medium of exchange, trade was necessarily limited to exchange of reciprocally desired goods, e.g. the trade between the hungry tailor and the ill-clad baker. Jevons called the situation where the supplier of good A is a demander of good B and vice versa a "double coincidence" of wants. A priori it appears that such double coincidences are rare even in the presence of market-clearing equilibrium prices that assure that for every buyer there is somewhere a willing seller.

> The earliest form of exchange must have consisted in giving what was not wanted directly for that which was wanted. This simple traffic we call barter . . . and distinguish it from sale and purchase in which one of the articles exchanged is intended to be held only for a short time, until it is parted with in a second act of exchange. The object which thus temporarily

intervenes in sale and purchase is money. At first sight it might seem that the use of money only doubles the trouble, by making two exchanges necessary where one was sufficient; but a slight analysis of the difficulties inherent in simple barter shows that the balance of trouble lies quite in the opposite direction . . . the first difficulty in barter is to find two persons whose disposable possessions mutually suit each other's wants. There may be many people wanting, and many possessing those things wanted; but to allow of an act of barter there must be a double coincidence, which will rarely happen.

The more likely event (i.e. absence of double coincidence of wants) is that for two traders, one of whom is the supplier of a good (good 1), the other demands, the latter's excess supplies – though of sufficient value at market prices to purchase the demand – are of a good (good 2) which the former does not require. Jevons argues that it is to overcome the absence of double coincidence of wants that monetary trade is introduced. The supplier of good 1, though apparently reluctant to accept good 2 in trade, will accept money. The supplier has less use for money than he had for good 2, but by common consent money can be traded directly for what the supplier of good 1 demands. Menger (1892) notes:

It is obvious . . . that a commodity should be given up by its owner in exchange for another more useful to him. But that every economic unit . . . should be willing to exchange his goods for little metal disks apparently useless as such, or for documents representing the latter, is a procedure so opposed to the ordinary course of things, that . . . [it is] downright "mysterious".

It takes an economy of at least three goods and at least three agents to generate a need for a medium of exchange. Three goods are needed since in a two-good economy, an individual's demand for one good implies, by budget balance, an equivalent supply of the other. Hence, in a two-good economy, the double coincidence condition is fulfilled and no need for a medium of exchange can arise. Three agents are required since, in a two-agent economy, market clearing implies the double coincidence condition and, hence again, there is no need for a medium of exchange. This simplicity is lost in a three-trader, three-good economy and a use for a medium of exchange arises. As Menger notes:

Even in the relatively simple . . . case, where an economic unit, A, requires a commodity possessed by B, and B requires one possessed by C, while C wants one that is owned by A – even here, under a rule of mere barter, the exchange of the goods in question would as a rule be of necessity left undone.

Formalizing this example, let the households be A, B, and C. Let the price vector, $p = (1, 1, 1)$. Let the household desired purchases be denoted x^A, x^B, and x^C and sales y^A, y^B, and y^C:

$$x^A = (1, 0, 0), \quad x^B = (0, 1, 0), \quad x^C = (0, 0, 1),$$

$$y^A = (0, 1, 0), \quad y^B = (0, 0, 1), \quad y^C = (1, 0, 0).$$

Note that this array constitutes a general equilibrium trading plan. For each demander there is a willing supplier. However, double coincidence of wants is not fulfilled. For every pair of agents there is no trade so that each agent can receive a good he demands in exchange for one he wishes to supply.

Suppose a zeroth good, money, also with a price of 1, is introduced and that the convention is adopted that money can be given in trade even when there is no excess supply and can be accepted though there is no excess demand. Then let any pair, e.g. A and B, meet to trade. Good 2 goes from A to B and a payment in money goes from B to A. This gives the new array:

$$x^A = (0, 1, 0, 0), \quad x^B = (1, 0, 0, 0), \quad x^C = (0, 0, 0, 1),$$

$$y^A = (1, 0, 0, 0), \quad y^B = (0, 0, 0, 1), \quad y^C = (0, 1, 0, 0).$$

Then let another pair trade, B and C for example. This gives the array:

$$x^A = (0, 1, 0, 0), \quad x^B = (0, 0, 0, 0), \quad x^C = (1, 0, 0, 0),$$

$$y^A = (1, 0, 0, 0), \quad y^B = (0, 0, 0, 0), \quad y^C = (0, 1, 0, 0).$$

Finally , C and A trade and all excess supplies and demands are reduced to nil.

The role for money as a medium of exchange derives from an overdeterminacy in the demand for goods in the trading process. When two agents trade, a supplier of one good is paid by delivery of goods of equal value from the buyer, quid pro quo. Goods are required as objects of consumption and as carriers of value to fulfill the quid pro quo. The second demand for goods derives from two conditions: the pairwiseness (or small group structure) of trade and the strategic requirement of the quid pro quo. Absent a medium of exchange, the overdeterminacy then implies that an equilibrium allocation cannot generally be implemented in a relatively short trading time using a pairwise decentralized trading process. It will require lengthy trade or significantly more complex organization. The alternative is to introduce a monetary commodity, providing an extra degree of freedom for the system to alleviate the overdeterminacy. The provision of money can allow revision of the trading process to permit implementation of the equilibrium allocation by decentralized pairwise trade in relatively short trading time.

The examples above treat a pure exchange economy with specialized endowments. In an economy with production, the counterpart of these examples is a tradition, going back to Adam Smith (1776), relating the role of money as a medium of exchange to the degree of specialization in production. An agent's output may be specialized, but his desired consumption is diverse, and is acquired through trade. If exchange were difficult, that difficulty would discourage specialization, by making it costly to implement in equilibrium; "division of labor is limited by the extent of the market [ease of trade]". A suitably acceptable, durable, and divisible good, that is a money, is required to even out transactions between agents whose desired trades with one another are non-synchronous or unequal in value.

> When the division of labour has been once thoroughly established, it is but a very small part of a man's wants, which the produce of his own labour can supply. He supplies the far greater part of them by exchanging that surplus part of the produce of his own labour, which is over and above his own consumption, for such parts of the produce of other men's labour as he has occasion for. Every man thus lives by exchanging. But when the division of labour first began to take place, this power of exchanging must frequently have been very much clogged and embarrassed in its operations . . . Every prudent man in every period of society after the first establishment of the division of labour, must naturally have endeavored to manage his affairs in such a manner, as to have at all times by him, besides the peculiar produce of his own industry, a certain quantity of some one commodity or other, such as he imagined few people would be likely to refuse in exchange for the produce of their industry.

Among the forms of specialization we expect to see in a low transaction cost economy is specialization in the transaction process itself. Specialists in trade include merchants, retailers, wholesalers, and financial intermediaries. Consistent with Smith's viewpoint, the distinct function of intermediary agents can be explained by scale economies in transaction costs. The use of intermediaries implies an increase in the gross volume of trade, since each commodity or security is traded several times. If scale economies on transaction costs are present, the savings associated with the concentration of transactions compensates for the added volume. A formal model of these views would use the transaction cost structure of Section 2 while characterizing transactions service firms with a non-convex technology. To model specialization in production activities, a fully articulated production sector with indivisibility in inputs (e.g. specialized labor) or other non-convexity would be appropriate. A general formal treatment of the Smithian view of the interaction of money and specialization is still absent from the literature.

3.2. A model of bilateral trade

First consider a static pure exchange economy (i.e. without production). The principal issues on the structure of bilateral trade can be posed in this model.

Let there be a fixed finite number of households H, with preferences represented by utility functions $U^h(c^h)$, $h = 1, \ldots, H$. Let

$$\begin{aligned} b^h &= \text{household } h\text{'s endowment vector, } b^h \in \boldsymbol{R}_+^N\,, \\ c^h &= \text{household } h\text{'s planned consumption vector, } c^h \in \boldsymbol{R}_+^N\,, \\ x^h &= \text{planned vector of net purchases by household } h,\ x^h \in \boldsymbol{R}_+^N\,, \\ y^h &= \text{planned vector of net sales by household } h,\ y^h \in \boldsymbol{R}_+^N\,, \\ p &= \text{vector of market prices, } p \in R_+^N\,. \end{aligned} \tag{6}$$

Prices are announced by an abstract market mechanism, sometimes personified as the Walrasian auctioneer, and are treated parametrically by households. Household h chooses c^h to

$$\max U^h(c^h) \tag{7}$$

subject to

$$p \cdot c^h = p \cdot b^h\,. \tag{8}$$

Then $x^h = [c^h - b^h]^+$ and $y^h = [b^h - c^h]^+$, where the $[\cdot]^+$ indicates the vector of non-negative elements of the argument. Prices, p, are said to be equilibrium prices when choosing c^h as in (7) above gives $\Sigma_h c^h = \Sigma_h b^h$, or equivalently:

$$\sum_h x^h = \sum_h y^h\,. \tag{9}$$

Note that

$$y^h \leq b^h \tag{10}$$

coördinate-wise, and

$$p \cdot x^h = p \cdot y^h . \tag{11}$$

An array $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$, fulfilling (6)–(11), can be called a general equilibrium trading plan. Our task now is to discover how the plan can be implemented.

There is a single essential revision of the general equilibrium model that ensures a role for a medium of exchange: replace the single budget constraint, (8) or (11), by a multiplicity of budget balance requirements, one for each bilateral trade. (8) and (11) require only that the total value of a household's purchases be paid for by the total proceeds of sales. (8) and (11) impose no restriction on individual transactions. The requirement that generates a demand for a medium of exchange is to require that sales pay for purchases *at each transaction.*

We denote this restriction as the quid pro quo constraint. Under bilateral trade, implementing an agent's trading plan, x^h and y^h, will involve many individual transactions, in most of which a planned purchase (a positive element of x^h) will not coincide with a planned sale (a positive element of y^h) of equal value. Fulfillment of the quid pro quo constraint will then require that the deficiency be satisfied by delivery of some good, either an ordinary commodity, or a specifically designated money.

The origins of the quid pro quo constraint are strategic [Ostroy (1973)]. The restriction is needed in a bilateral trade setting to ensure individual fulfillment of the budget constraint (8) or (11). Without the quid pro quo restriction applied to individual transactions there might be no effective means to prevent violation of budget constraint and a resultant shortage of some good at the completion of trade.

We will take the trading period to be divided into a large countable number of instants suitable for trade. At each instant each trader can trade with at most one other trader, pairwise. Denote an arbitrary finite schedule in which each trader meets each other trader precisely once (disjoint pairs meeting simultaneously) as a *round.*

Let the number of trading instants in a round be K. K must be at least as large as the number of agents minus one. Trading instants are denoted $k = 1, \ldots, K$. At the start of the kth instant trader i's deliverable supplies will be represented by w_i^k. $w_i^1 = b_i$. The change in i's holdings between k and $k+1$, $a_i^k = w_i^{k+1} - \omega_i^k$, is the trade i performs in instant k. Trader i's hitherto unsatisfied excess demands on entering k are $v_i^k = v_i^1 - \sum_{\kappa=1}^{\kappa=k-1} a_i^\kappa$. $v_i^1 \equiv x^i - y^i$.

Consider the meeting and trade between i and j at instant k. Each brings his holdings w_i^k and w_j^k, to the pair. Positive entries in the vector a_i^k indicate goods going from j to i and negative entries, goods going from i to j. After trading, i's holdings will be $w_i^{k+1} = w_i^k + a_i^k$ and j's will be $w_j^{k+1} = w_j^k + a_j^k$. We place the following three restrictions on a_i^k, a_j^k:

(A.1) $w_i^k + a_i^k \geq 0,\ w_i^k + a_i^k \geq 0$ (non-negativity of holdings) ,

(A.2) $a_i^k = -a_j^k$ (pairwise trade) ,

(A.3) $p \cdot a_i^k = 0 = p \cdot a_j^k$ (the quid pro quo) .

Should trades fulfill (A) for all $i, j \in I$ and $k = 1, \ldots, K$, we shall say that the sequence of trades is admissible.

The non-negativity requirement, (A.1), says that a trader can at no time have a negative holding of any commodity. A trader cannot deliver in trade more of a commodity than he currently holds. This may be interpreted as a prohibition on the issue of I.O.U.s.

The pairwiseness condition, (A.2), says that in the process of trade, goods delivered are received, and vice versa. To the extent that goods are lost in storage or used up in transactions costs, this occurs in a separate process.

The quid pro quo condition, (A.3), requires that in the trade between i and j, each delivers to the other goods of equal value. Full payment is made for value received where goods are evaluated at equilibrium prices.

Conditions (A.1) and (A.2) are feasibility restrictions defining bilateral exchange. The origins of (A.3) are strategic; it is needed to enforce the budget constraint (8) or (11).

Given prices, an order of meetings for the pairs of traders, and an admissible sequence of trades, the outcome can be described as the resulting allocation of goods among traders. At the end of one round the outcome for trader i is $\sum_{k=1}^{K} a_i^k$. We will say that full execution of excess demands has been achieved in one round if

(E) $$\sum_{k=1}^{k=K} a_i^k = x^i - y^i\ , \quad \forall i\ .$$

Should time run out ($k = K$) before all demands are fulfilled and supplies delivered, (E) will not be satisfied.

Will (E) be fulfilled without violating (A)? To answer this question we need a model of how trading decisions are made. We will characterize the trading decision of pair i, j, the trading rule, as a function of i, j's current holdings w_i^k, w_j^k, and other information, $L_{i,j}^k$. The current holdings define the set of trades possible consistent with (A). Other information allows them to choose among the possibilities.

Define a trading rule as a function $\rho(w_i^k, w_j^k \mid L_{i,j}^k) = (a_i^k, a_j^k)$, where $L_{i,j}^k$ is the set of information, beyond their current holdings, available to the pair at instant k. An economic arrangement is generally described as decentralized if it involves individual agents making decisions to further their individual aims based on a fairly small body of universally communicated information (e.g. prices) and on information which the agents themselves may be supposed to possess (e.g. individual tastes and endowments).

Hence, we say a trading rule is *decentralized* if

(D.1) $$L_{i,j}^k = \{(v_i^k, v_j^k), p\},$$

or

(D.2) $$L_{i,j}^k = \{(v_i^k, v_j^k), (i, j), p\}.$$

Conversely, a non-decentralized trading procedure may be characterized by

(C) $$L_{i,j}^k = \{(v_1^k, v_2^k, \ldots, v_H^k), (i, j), p\}.$$

(D.1) describes as decentralized a rule that formulates a pair's trade by using prices, the pair's current excess demands and supplies. (D.1) is anonymous; (D.2) allows the rule to use the names of the agents as well. (C) represents as non-decentralized a procedure that requires information on all agents' excess demands in order to formulate the trade for any pair of agents. The informational requirements of (C) are thought to be sufficiently great as to imply centralization in the collection and dissemination of the information, and in implementation of the rule. A rule that actually makes full use of (C) would require traders to make trades based on the excess demands of other traders with whom they may have no remaining opportunity to trade in the balance of the round.

Two weaker concepts of full execution are useful. In particular, we wish to consider a trading process that requires more than one round of meetings between pairs of traders to arrange for contracting and payment of obligations. Another approaches full execution as a limit after a multiplicity of repeated trading opportunities. These alternatives will allow us to discuss the trade-off between trading time and trading organization. Hence, we say that full execution is achieved in three rounds if, for each i:

(E.3) $$\sum_{k=1}^{k=3K} a_i^k = x^i - y^i.$$

We now seek to describe convergence to full execution as the limit of a trading process. Let ψ^ν be a sequence of reals, $1 \geq \psi^\nu \geq 0$, with $\psi^\nu \to 0$. Then we say that full execution is approached as ψ^ν if

$$(\mathrm{E}.\psi^\nu) \qquad \sum_i p \cdot [v^{1+\nu k}]_+ \leq \psi^\nu \sum_i p \cdot x^i .$$

That is, we measure execution by the proportion of the value of demands that is fulfilled. By the end of the νth round, suppose at least the proportion $1 - \psi^\nu$ of the original value of excess demand has been fulfilled. Then we say that trade converges to full execution as ψ^ν.

In actual trading processes, certain commodities and agents enter with distinct asymmetric functions differing from those of other goods or agents. We will find it useful to distinguish money as a special commodity. Among traders we will distinguish one that acts as a bank.

The distinctive element of actual monetary economies is that almost all transactions have as one side a financial instrument thought of as "money". In order for successful monetary trade to take place without violating (A.1), non-negativity, agents must have, at each trading instant, sufficient money to finance their current purchases. The money will come from endowment or the proceeds of past sales. We are interested then in characterizing economies with sufficient endowment of money so that illiquidity due to exhaustion of money holdings in the course of trade need not be a problem. In actual economies, this is ensured partly through purposeful timing of transactions (one goes to the bank for cash before buying lunch), but direct treatment of the timing decision would introduce greater complexity than we wish to treat in this model. Hence, we will characterize, at least at first, a monetary economy as one endowed with a sufficient stock of a monetary commodity to be used as medium of exchange. It must be distributed sufficiently broadly in sufficiently great quantity (in value terms) among the holders that all agents find that they can finance all desired purchases from endowment of the money commodity. This is obviously too strong a requirement to be taken seriously as a primitive description of an actual economy. Rather, it reflects the array of holdings that agents may arrange at the start of the trading process, to facilitate the subsequent process of trade. The alternative, developed later below is to describe a bank as an institution that creates monetary credit instruments, hence overcoming shortage of monetary commodity. Both of these approaches ignore – under the assumption that it is too complex to model – explicit timing decisions of individual agents designed to assure continuous liquidity as needed. This requirement enters essentially, however, in the analysis of sequence economies with transactions costs.

Definition (Monetary economy). The economy is said to be monetary if there is a good, 0, so that for all households h,

$$p_0 \cdot b_0^h \geq p \cdot x^h .$$

A monetary economy is hence described here by the property that there is a zeroth good universally held in a quantity sufficient to finance all purchases. In the trading rule used to demonstrate the superiority (decentralizability) of monetary trade, the zeroth good enters (like money) essentially asymmetrically in the exchange process.

Definition (Bank credit economy). The economy is said to be a bank credit economy, if there are goods d (debt), and c (banknotes), for which the non-negativity requirement (A.1) is waived for trades between households and the bank.

A bank is defined here as a trader that can buy household debt and issue its own debt instruments. Debt necessarily involves negative holdings of the debt instrument by its issuer. Hence, the definition describes a limited violation of (A.1). This means that the bank is allowed to extend credit (contrary to the idea of informational decentralization) in a way that other traders are forbidden.

The role of money as a medium of exchange consists in allowing full execution to be achieved expeditiously (in one round) by a decentralized rule, whereas in the absence of money, full execution requires more time, ample goods inventories to act as trading stocks, or sufficient information to support a non-decentralized rule. These results are embodied in the following theorems.

Theorem 1. *Let* $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$ *be a general equilibrium trading plan. Then there is a trading rule satisfying* (A), (C) *and* (E).

Theorem 2. *There is no trading rule that, for all general equilibrium trading plans* $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$, *fulfills* (A), (E), *and* (D.2) [*or* (D.1)].

Theorem 3. *Let* $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$ *be a general equilibrium trading plan of a monetary economy. Then there is a trading rule fulfilling* (A), (E), *and* (D.1).

Theorem 4. *Let* $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$ *be a general equilibrium trading plan of a bank credit economy. Then there is a trading rule fulfilling* (A), (E.3), *and* (D.2).

Theorem 5. *Let* $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$ *be a general equilibrium trading plan. Then there is a trading rule fulfilling* (A), (E.$(\frac{1}{2})^{\nu}$), *and* (D.1).

Theorem 6. *Let* $(p, \langle x^h \rangle_h, \langle y^h \rangle_h, \langle b^h \rangle_h)$ *be a general equilibrium trading plan. For some agent* h^*, *let* $b^{h^*} \geq \sum_{h \neq h^*} x^h$ *(the inequality holds co-ordinatewise). Then there is a trading rule fulfilling* (A), (E), *and* (D.2).

Theorems 1 and 2 demonstrate the trade-off between full execution and limited information. Together, they say that although there exists a rule that makes (A) and (E) compatible for every general equilibrium, that rule must be centralized. Theorem 3 says that if there is a commodity such that the value of each trader's holdings of it is at least equal to the value of his planned purchases of all other commodities, then decentralized trading is compatible with full execution. In particular, the commodity 0 in Theorem 3 is regarded as money, and it behaves as money in the trading rule used to prove that theorem.

Theorem 4 applies to an economy with a bank. If the bank provides sufficient credit instruments, then the decentralized trade suggested in Theorem 3 can occur with the credit instruments acting as money, though additional time for financial transactions may be required. Theorem 5 says that, absent the money or credit of Theorems 3 and 4, decentralized trade can converge to full execution over many trading rounds. It will converge geometrically over time, but there is apparently no guarantee of achieving full execution in finite time. Theorem 6 says that the presence of a trader with sufficient trading stocks to act as a clearing-house allows full execution to be achieved in one round of decentralized trade.

To summarize, in a bilateral trading model we have the following results:

(i) It is not generally possible to implement a general equilibrium trading plan in one round in a decentralized fashion without money, bank credit, or large trading inventories (Theorem 2).

(ii) Implementation is possible without money or credit using non-decentralized trading procedures (Theorem 1), in a decentralized fashion requiring much more than one round (Theorem 5), or in one round of decentralized trade if there are ample trading inventories (Theorem 6).

(iii) Monetary trade using money or credit instruments allows decentralized implementation of the allocation in one round. In the case of the bank credit model some extra trading time may be needed to arrange and repay credit (Theorems 3 and 4).

Points (i) and (ii) summarize the inconvenience of barter. Point (iii) says that money allows rapid decentralized implementation of a general equilibrium trading plan.

3.3. Discussion of proofs of Theorems 1–6

The proof of Theorem 1, existence of a centralized procedure, is by construction and comes in two parts. First it is shown that the complex of excess demands and supplies can always be decomposed into a finite number of elementary configurations (chains) so that each agent in the chain has an excess demand for one good, excess supply of another and, for each good, supply equals demand across the chain. A chain can be represented as shown in Figure 1.1. This is read: "A has an excess supply of 1 for which B has an excess

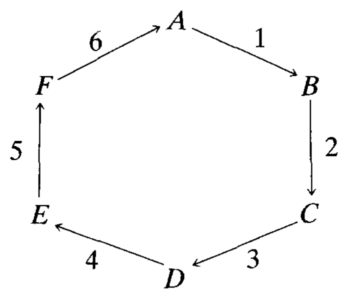

Figure 1.1

demand; B has an excess supply of 2 for which C has an excess demand; . . . 6 for which A has an excess demand". A centralized trading procedure is developed for trades in chains. The centralized procedure for the economy then is for each pair of traders, when they meet to trade, to perform the sum of the trades appropriate to the chains they have in common.

We wish to show that each chain can have its demands fulfilled in a single sequence of trades. The trading procedure that achieves this is simple to state but requires sufficient information and coordination to allocate traders to chains and to let them know what chains they have in common. Hence, it does not qualify as decentralized. When two traders meet, if they are members of a common chain they exchange excess supplies corresponding to the chain. This breaks the chain into two smaller disjoint chains. The process continues until all chains are of unit length, i.e. excess demands are fulfilled. This can be illustrated diagrammatically. Suppose A and C are the first elements of the chain to meet. They exchange excess supplies. The resulting array is Figure 1.2, i.e. two smaller disjoint chains. The process is repeated for each chain separately until each excess demand is fulfilled.

This complexity is unavoidable. Theorem 2 says that it is not generally possible to find a decentralized trading rule that moves in limited time to the equilibrium allocation. The proof consists of presenting two different economies with some traders identical in each. The example is set up so that

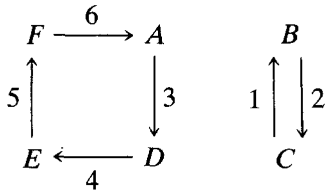

Figure 1.2

the trades these traders must make in one economy to achieve equilibrium consistent with other traders' demands would preclude achievement of equilibrium in the other. Under a decentralized rule, however, the traders cannot distinguish between the two for purposes of deciding on their trades, since the difference is not in their own excess demands but in others'. Hence, full execution is not generally decentralizable.

In the proof of Theorem 3, commodity 0 acts as the medium of exchange. When two traders meet, commodities of which one has an excess supply and the other has an excess demand go from the supplier to the demander until the supply or the demand is exhausted. Failure of the quid pro quo is made up by trade in 0. The procedure is decentralized since each pair needs only information on its own excess demands and supplies in order to make an intelligent trading decision. By definition of a monetary economy, there is sufficient stock of good 0 so that the procedure may be followed without violating (A.1), non-negativity.

The proof of Theorem 4 simply requires that credit arrangements be made with the bank in the first round so that each agent has banknotes at the beginning of the second round sufficient to finance all of his planned purchases in the round. Theorem 3 is then applied to the resulting array. In the third round the only activity is repayment of bank credit.

In the proof of Theorem 5 all goods act indiscriminately as a means of payment in a trading rule that otherwise follows the rule in Theorem 3. It can be shown that by the end of one round, at least half the initial demands are satisfied. Unfortunately, goods acting as a means of payment may end the first round in the hands of those who do not demand them. The procedure is repeated in the second round, so that outstanding demands are reduced to a fourth of their initial level, and so forth.

In the proof of Theorem 6, agent h^* acts as a clearing-house. His large endowment allows him successfully to fulfill this function.

What do we conclude from the analysis of this section? Essentially, that implementing a general equilibrium allocation by bilateral trade is a tricky proposition, one made significantly easier by the introduction of a single medium of exchange, in ample quantity, entering asymmetrically in the trading process. The trade-offs presented in this problem are among trading time, informational decentralization, and inventories. Sufficient slack in any one of these three variables can make up for stringency in the other two. When all three are at the minimal levels consistent with a general equilibrium plan, it is not generally possible to organize pairwise trade to the equilibrium allocation.

Pairwiseness is associated with three restrictions on the structure of trade:

(1) The quid pro quo; a multiplicity of pairwise budget constraints.

(2) Non-negativity; traders in bilateral transactions can deliver only goods they have.

(3) Informational decentralization; the information or coordination needed to implement trades must be available pairwise.

These conditions are familiar; they are fulfilled by the individual's single trade with the market in the Arrow–Debreu model. But here they are applied to each pairwise trade. The multiplicity of bilateral trades – and their interdependence – implies, however, that extending these conditions to the bilateral trading model is not innocuous. It so overdetermines the system that – in contrast to the Arrow–Debreu general equilibrium model – expeditious trade to equilibrum is not generally possible. This represents the inconveniences of barter. The introduction of money as a medium of exchange, in sufficient quantity to avoid the non-negativity constraint binding, restores sufficient flexibility to allow all three conditions to be fulfilled and to allow trade to proceed to equilibrium. Hence, the superiority of monetary trade.

3.4. The spontaneous emergence of media of exchange

It is useful to divide the elementary issues concerning monetary exchange into two separate questions:

- What is the function of a common medium of exchange?
- Why have certain commodities become media of exchange?

With regard to the first question, we have seen that in both the bilateral trade model above and in the transactions costs variant of the multilateral Arrow–Debreu model of the previous section, the function of a common medium of exchange is to enforce the accounting identity budget constraint in a sequential trading environment. With regard to the second question, it is common to provide a checklist of desirable characteristics such as portability, divisibility, and durability. In this chapter we omit any discussion of the characteristics of a money commodity while implicitly assuming that it has all of the properties necessary to fulfill its role as a medium of exchange. There is, however, a related question

- How have certain commodities come to be used as media of exchange?

Jones (1976) addressed this question from a point of view first stressed by Menger. Menger (1892) claimed that money need not be a creature of the state, but could arise spontaneously through the market behavior of individuals.

The setting is again one of barter through bilateral exchange. Each individual has an excess demand vector of the form:

$$(0, \ldots, -1, \ldots, 1, \ldots),$$

i.e. there is one commodity to be purchased and one to be sold. The aggregate pattern of excess demands among the population is such that at the one-for-one exchange rate aggregate excess demands for each commodity is zero.

The quid pro quo is taken for granted. The question is: How will it be imposed? Will it be through direct barter where individuals accept as payment for goods sold only what they wish ultimately to consume, or will it be through indirect barter? And if through indirect barter, is there a pattern in which one (or a few) goods emerge as the unique medium of indirect trade? Note that the monetary pattern of exchange amounts to a replacement of the traditional double coincidence of wants by an indirect double-coincidence that individuals spontaneously recognize as less costly to achieve.

Subadditivity in transactions costs

If indirect exchange is to be less costly than direct, the cost of exchanging commodity i for j will have to be greater than the cost of exchanging i for k and for exchanging k for j. Suppose c_{ij} and c_{ik} are the costs of exchanging i for j and i for k, respectively. If these costs are additive in the sense that they can be additively decomposed into

$$c_{ij} = c_i^s + c_j^b,$$

where c_i^s is the selling cost of i and c_j^b is the buying cost of j, then

$$c_{ij} < c_{ik} + c_{kj} = c_i^s + c_k^b + c_k^s + c_j^b.$$

The immediate conclusion is the opposite of what we are seeking: direct barter is less costly than any indirect barter. Because arguments based on the physical characteristics of commodities or on transport costs typically have this additive form, they will have a limited role in explaining the function of a medium of exchange as well as its spontaneous emergence. As in the model of the previous section, we therefore ignore variations in the physical properties of commodities and assume that they are all indistinguishable.

It is in the search or time costs of exchange that Jones finds the subadditivity essential to an explanation of indirect trade. Let p_i be the probability that a trader will wish to buy or sell commodity i. There is no correlation between the commodity a trader wishes to buy and the one he wishes to sell. Thus, $p_i p_j$ represents the probability that a trader will wish to exchange i for j.

In this market with random pairwise meeting among traders, a trader wishing to exchange i for j who refuses to make any indirect exchanges must expect that the number of meetings required to make such an exchange will be

$$\frac{1}{p_i p_j}\,.$$

Adopting a two-stage strategy of trading i for k and then k for j, the expected number of meetings will be

$$\frac{1}{p_i p_k} + \frac{1}{p_k p_j}\,.$$

Therefore, if $p_k > p_i + p_j$, there is subadditivity in the time costs of exchange since

$$\frac{1}{p_i p_j} > \frac{1}{p_i p_k} + \frac{1}{p_k p_j}\,.$$

The logic of this argument implies that if indirect trade is advantageous, the most desirable course of action is to use a commodity n for which $p_n = \max\{p_k\}$, a commodity that is most frequently bought and sold.

Jones confined his attention to *unconditional* trading strategies in which a plan to trade i for k and then k for j is maintained even if the trader happens to meet someone willing to trade i for j directly before he meets someone willing to trade i for k. Recently, Oh (1989) has provided a revised version with more flexible and rational *conditional* trading strategies. With these strategies, Oh demonstrates that the optimal strategy for any individual is to exchange the good he has either for the good he ultimately wants or for any other with a higher probability of being traded. It follows from this observation that if $p_n > p_i$ for $i \neq n$, all the individuals not endowed with good n will use it as a generally acceptable medium of exchange; and, provided that the gains from making a chain of indirect trades is undertaken only if it promises more than some sufficiently small expected gain, most of the indirect trade is confined to the good n. Thus, a good that is commonly known to be more frequently traded becomes the essentially unique medium of indirect exchange.

4. The consequences of budget enforcement for the allocation of resources

The term "dichotomy" is used in monetary theory to indicate the separation between the real and the monetary sectors of the economy. It refers to the dichotomous manner in which relative and nominal prices are determined;

relative prices are determined in the real sector of the economy and nominal prices are determined in the monetary sector. (See Subsection 5.1 for a definition of the *classical dichotomy*.) The models of the transactions role of money described so far recapitulate the dichotomy tradition.

For example, in the models of the previous section the timing element was limited to the issue of how rapidly full execution of what was more or less a given static equilibrium could be achieved. The role of money as a record-keeping device or as a medium of indirect exchange was confined to demonstrations of the minimality of the time to execute those trades subject to the constraints of sequential bilateral exchange and the quid pro quo. We might conclude that after demonstrating the rationale of monetary trade as an aid in executing exchange, we are free to regard the execution problem as solved and return to the frictionless theory of value for the determinants of real allocations.

A similar lesson can also be drawn from the contents of Section 2. Although it was certainly necessary to introduce transactions costs in the Arrow–Debreu theory, the role that money played encourages a dichotomous treatment of the model. First, recognize that money functions to undo the sequence of budget constraints that would otherwise restrict equilibrium allocations. Then point out that once money is introduced the model can be solved by looking only at its "real" features, which may include transactions costs.

For the remainder of this chapter instead of concentrating on the transactions role of money without any special consideration as to how this affects the allocation of resources, we change our focus to admit some of the consequences for "real" theory of the budget enforcement problem. In Subsection 4.1 we study a simple model due to Townsend (1980) in which the enforcement problem has at least a marginal impact unless some rather idealized government intervention is pursued. In Subsection 4.2 we analyze a stochastic variant of the model of Subsection 4.1 due to Green (1987) in which the enforcement problem fundamentally alters the allocation of resources.

4.1. Another version of the "Pair of Robinson Crusoes"

In our story of the two Robinson Crusoes exchanging dinners, the nature of the exchange precluded double coincidence at any trading date. Let us consider a related problem analyzed by Townsend (1980).

There are two *types* of individuals, type A and type B. Their endowments, w_t^{A} and w_t^{B}, in period t alternate according to the following simple scheme:

$$w_t^{\mathrm{A}} = \begin{cases} 0\,, & t \text{ even}\,, \\ 1\,, & t \text{ odd}\,; \end{cases}$$

$$w_t^{\mathrm{B}} = \begin{cases} 1\,, & t \text{ even}\,, \\ 0\,, & t \text{ odd}\,. \end{cases}$$

Each type evaluates the intertemporal consumption stream $c = (c_0, c_1, \ldots, c_t, \ldots)$ according to the same utility function:

$$U(c) = \sum_{t \in T} \beta^t u(c_t)\,,$$

where $T = \{0, 1, \ldots, t, \ldots\}$. The single period utility function $u(\cdot)$ is strictly concave, differentiable and increasing on $\boldsymbol{R}_+$.

The gains from trade arise from the smoothing of consumptions over time compared to the "feast or famine" pattern of initial endowments. Commodities are assumed to be perishable. This smoothing is made possible by assuming that an individual of type A always meets an individual of type B in every period so that there is always an opportunity to "exchange". Suppose that an A and B, once paired, never meet again. Then the obligation to give when you have a positive endowment might be regarded as part of a social contract between each individual and the rest of society, rather than an exchange between a pair of individuals. Indeed, such a contract might be enforceable, since it would meet the necessary if not sufficient condition that there is common knowledge between any pair when one of them does not live up to the social agreement. Alternatively, such an agreement could be described in terms of the sequence economy model of Section 2 as one in which there are forward markets for the sale and delivery of future commodities: sell today while simultaneously buying forward for tomorrow. To forestall these possibilities, assume that once the individuals begin to trade they are on their own without recourse to a higher authority, or alternatively that the costs of making forward transactions are prohibitive.

With the myopic perspective of spot trading only without any enforcement mechanism, the complete absence of a double coincidence of wants at each date leads to the conclusion that there will be no trading. Into this impasse, Townsend introduces a transferable asset, without any utility of its own, called "money" to encourage exchange. Is it serving the same role as the money described in the previous models of exchange? Certainly, it helps to facilitate transactions; but, more specifically, it does this through the familiar device of undoing the inhibitions on trade caused by the sequence-of-budget-constraints problem.

In several respects this model can be regarded as a special case of the more general sequence economy of Section 2. However, it does go beyond the finite end-point assumption of the sequence economy model (and the bilateral exchange models of Section 3) by introducing an infinitely receding time horizon. Following Townsend, we examine the implications for the allocation of resources of this elementary model with a money budget constraint.

With money, the budget constraint in each period becomes:

$$m_{t+1} = p_t(w_t - c_t) + m_t \geq 0\,, \quad t \in T\,,$$

where m_{t+1} is the money carried over from period t and p_t is the price of the commodity at date t in terms of money. In addition, of course, $m_0 \geq 0$.

The fact that an individual's money balances must always be non-negative means that the value of his current excess demands $\max\{p_t(c_t - w_t), 0\}$ cannot exceed his money balances at the start of the period m_t. With a single commodity, current purchases will be positive only when sales are zero, so the money budget constraint is effectively a cash-in-advance constraint, i.e. $p_t(w_t - c_t) < 0$ implies $w_t = 0$, and therefore since $m_{t+1} \geq 0$ we must have:

$$m_t \geq p_t c_t\,.$$

Maximizing the intertemporal utility function U given above subject to the money budget constraints yields the following first-order conditions for a maximum:

$$-\beta^{t-1}\frac{u'(c_{t-1})}{p_{t-1}} + \beta^t\frac{u'(c^t)}{p_t} + \theta_t = 0\,,$$

where θ_t is the Lagrange multiplier associated with the tth budget constraint.

To interpret this condition, if an extra dollar is transferred from consumption on c_{t-1} to c_t the individual will have to give up c_{t-1}/p_{t-1} units of consumption at $t-1$ to obtain c_t/p_t units of consumption at t, which at the margin leads to a loss of utility of

$$\beta^{t-1}\frac{u'(c_{t-1})}{p_{t-1}}$$

compared to the gain of

$$\beta^t\frac{u'(c_t)}{p_t}\,.$$

If $\theta_t = 0$, the loss balances the gain and

$$\frac{u'(c_{t-1})}{\beta u'(c_t)} = \frac{p_{t-1}}{p_t}\,.$$

This is the condition that would prevail in an intertemporal optimization problem with the single intertemporal budget constraint of the form:

$$(*) \qquad \sum_{t \in T} p_t(w_t - c_t) = 0 \, .$$

It is also the necessary condition for a Pareto-optimal allocation in this economy where an individual of type A always meets an individual of type B. However, as long as $\theta_t > 0$, the first-order conditions from the single budget constraint problem and the Pareto-efficient allocation are necessarily violated.

Because θ_t is the Lagrange multiplier for the tth constraint, we have:

$$\theta_t \cdot m_t = 0 \, ,$$

i.e. the tth constraint is only binding when $m_t = 0$. Is there a monetary equilibrium in which the money budget constraint is never binding, i.e. a monetary equilibrium in which money is a veil allowing the frictionless Pareto-efficient barter allocation to be realized?

This question is similar to that of the optimal quantity of money treated elsewhere in this Handbook. It is readily concluded that for efficiency, prices must be decreasing over time at the rate $1 - \beta$ and Townsend shows that if the monetary budget constraint is to be non-binding this will require at least one of the individuals to be accumulating money balances over time in contradiction to the maximizing hypothesis of equilibrium. With lump-sum taxes on money balances, it is possible to construct a monetary equilibrium with effectively non-binding budget constraints through a government engineered deflation. However, such an equilibrium requires specific information about which individual to tax in each period and therefore violates the anonymity that would presumably underlie a decentralized description of equilibrium. Townsend also shows that there do exist monetary equilibria without any lump-sum taxes. They are Pareto superior to autarchy but not Pareto optimal.

4.2. Record-keeping in the chicken–egg economy

From the model of the previous subsection we could conclude that despite the absence of forward markets, the introduction of money could, under certain government policies, be a "veil" for the frictionless barter economy in the sense that the budget enforcement problem could be resolved without any loss in efficiency. Here we describe a model in which money, or any other mechanism for coping with the budget enforcement problem, is never a veil.

Amend the description of the sequence of endowments in the model of

Subsection 4.1 so that in each period an individual has a 50–50 chance of having an endowment of 0 to 1. (In the parable of Subsection 1.3, the endowment refers to whether or not one's chicken laid an egg.) Let $c = (c_1, \ldots, c_t, \ldots)$ be a consumption sequence, where $c_t \in \boldsymbol{R}$ is modified to include the possibility of negative consumption! (This is to eliminate the complications that would follow from boundary conditions on consumption.)

Without the benefit of trade the utility of an individual's initial endowment, the random sequence ω, is:

$$EU(\omega) = \sum_{t=0}^{t=\infty} \frac{\beta^t[u(0) + u(1)]}{2} = \frac{[u(0) + u(1)]}{2(1-\beta)} .$$

In this model the gains from trade are due entirely to risk-pooling. With a large number of individuals, literally a continuum, the per capita endowment would be 1/2 with probability one. This would permit each individual to exchange his random variable ω for the perfectly certain consumption stream $c^* = (1/2, 1/2, 1/2, \ldots)$. Assuming as we shall that u: $\boldsymbol{R} \rightarrow \boldsymbol{R}$ is concave (to reflect risk aversion) and increasing, then

$$EU(\omega) < EU(c^*) = \sum_{t \in T} \beta^t u(1/2) = \frac{u(1/2)}{(1-\beta)} .$$

It is evident that if each individual were to receive the consumption stream c^*, this would be a Pareto-efficient allocation for the economy as a whole. [Of course, this would not be the only Pareto-efficient allocation. Let the set of individuals be I and λ a population measure on I with $\lambda(I) = 1$. Then any allocation $c(i) = (\alpha(i), \alpha(i), \alpha(i), \ldots)$ in which each individual receives a constant stream of consumption and

$$\int \alpha(i)\, \mathrm{d}\lambda(i) = 1/2$$

would also be Pareto efficient.]

The stream c^* can be singled out as the symmetric or equal treatment allocation for the individuals who are in fact ex ante identical. It is the obvious candidate for equilibrium among the Pareto-efficient allocations. If information about the realizations of individual endowments were public, then c^* could be achieved as a competitive equilibrium through the use of forward/contingent contracts [Arrow (1964) and Debreu (1959)].

To describe this equilibrium, let $s^t = (s_0, s_1, \ldots, s_t) \in \{0, 1\}^{t+1}$ be a realization of an individual's random endowment – called an event – up to and including date t. Thus, $\omega_t(s^t) = s_t$ is the individual's endowment at t in the event s^t. For each event s^t there is an event-contingent price $p_t(s^t)$ for commodities.

(A full description of an event should include the realization of endowments for all individuals, but that can conveniently be ignored here.)

Once commodities and prices are refined to be event contingent, we can construct the all-important budget constraint as:

$$(**) \qquad \sum_{t\in T} \sum_{s^t\in\{0,1\}^{t+1}} p_t(s^t)[c_t(s^t) - \omega_t(s^t)] = 0 ,$$

where $c_t(s^t)$ is the consumption choice at t contingent on s^t and $z_t(s^t)$ is the net trade at s^t. The budget constraint $(**)$ is the state-contingent analog of $(*)$ in Subsection 4.1. Of course, this does not imply *for any particular realization of the individual's endowment* that realized purchases will equal realized sales.

It readily follows that setting $p_t(s^t) = \beta^t$, $t \in T$, the expected value of the individual's lifetime endowment is $[2(1-\beta)]^{-1}$. If the individual were to set $c_t(s^t) = 1/2$, all s^t and all $t \in T$, then $(**)$ would be satisfied. At these prices, such a trading plan would maximize expected utility $EU(c)$ subject to the budget constraint and, with a continuum of individuals, markets would clear with probability 1.

The analysis is rather different when information about the realization of an individual's endowment is private. Radner (1968) was one of the first to call attention to the difficulties for contingent commodity analysis when information about events is not common knowledge. With different information among individuals about an event, it would be difficult to write enforceable event-contingent contracts. The equilibrium trading arrangement under common information *could* be carried out under private information. Each time an individual's chicken does not lay, the individual could request 1/2 an egg and each time it does, the individual could offer 1/2 an egg. But without an omniscient enforcement mechanism, narrow self-interest would argue the outcome would be otherwise. (Recall the similar problem in the model of bilateral exchange in Subsection 3.1.)

Radner's solution was to limit event-contingent exchanges to those that were common knowledge. If we followed this prescription while assuming that information was private, i.e. each individual was the only one who knew whether his chicken had laid an egg, there would be no trade since no event would be common to more than one individual. Since Radner's work, there has been considerable progress in the analysis of moral hazard problems and in the literature on incentives as a whole. Green (1987) has made a contribution to this field, and one particularly relevant for monetary theory, by posing and then answering the question: What is an efficient incentive-compatible trading arrangement for this economy?

With private information it is evident that incentive compatibility will preclude the achievement of a trading arrangement yielding expected utility $EU(c^*)$. In the equilibrium trading plan $z^* = (z_t^*(s^t))$, net trades at t are independent of the history of the event before t, i.e.

$$z_t^*(s_0, s_1, \ldots, s_{t-1}, s_t) = z_t^*(s_0', s_1', \ldots, s_{t-1}', s_t) .$$

Indeed, this is the key to providing complete risk-pooling. However, any incentive-compatible trading arrangement with private information must necessarily be *dependent* on the realization of one's endowment. Since any such dependency is incompatible with full efficiency, *the conditions for incentive compatibility and full efficiency are therefore disjoint* [see Taub (1988)].

This conclusion goes beyond the model of Subsection 4.1 where it was shown that with certain policies there was an optimum quantity of money that would overcome the obstacles posed by the sequence of money budget constraints so that the economy could reach an allocation which overcame the budget enforcement problem. Here the first-best allocations under common information about events are simply unachievable as an equilibrium under any trading arrangement when there is private information. We do not point to this as an instance of "market failure" in the sense that there are potentially corrective actions by the government that would lead to a fully efficient allocation. Rather, we see this as an illustration of one of the essential differences between frictionless barter model and a more decentralized environment in which the record-keeping function of a trading arrangement has an essential role to play: in a decentralized economy the record-keeping function of money imposes a binding constraint on trade.

With private information, how would trade take place? Green (1987) provides an idealized description. Imagine competition among "banks" to operate a trading arrangement for the economy. The winner will be the bank offering the best arrangement in the sense of providing the highest expected utility to individuals subject to the condition that the bank does not lose money, a condition necessarily including a proviso that the arrangement be incentive compatible.

The arrangement, or contract, is a sequence of functions:

$$z_t \colon \{0, 1\}^{t+1} \to R , \quad t = 0, 1, 2, \ldots .$$

The quantity $z_t(s^t)$ gives the net purchase at t to which an individual whose endowment history is s^t is entitled. (If $z_t < 0$, the individual is called upon to supply that many units of the commodity.) These functions must be chosen to encourage individuals always to tell the truth.

Let $r^t \colon \{0, 1\}^{t+1} \to \{0, 1\}^{t+1}$, $t \in T$, be a sequence of functions describing the reported endowment histories for an individual as a function of the actual endowment histories, or events. The fact that all endowment histories are private information means that the bank must rely on reports rather than events.

The consequence for an individual who adopts $r = \{r^t\}$ when the bank offers the contract $z = \{z_t(r^t)\}$ is the expected utility

$$EU(z \mid r) = \sum_{t \in T} \beta^t E_{s^t}[u(\omega_t(s^t) + z_t(r^t(s^t)))] .$$

For the contract z to be incentive compatible, it must encourage individuals always to report the events, i.e. for all reporting strategies r_0'

$$EU(z \mid s) \geq EU(z \mid r) .$$

The arrangement must also be feasible for the economy as a whole. Here Green takes a certain liberty by allowing the bank itself to face a single budget constraint over time rather than a sequence of constraints, one at each date. The bank can take β units of consumption at t and transform it into one unit of consumption at $t+1$, and vice versa. The conjunction of the feasibility and incentive-compatibility conditions define the optimal contract $z = (z_t(s^t))$.

To emphasize the distinctions between the trading arrangement z^* with public information and forward/contingent contracts and the optimal incentive contract z when information is private, it is useful to recall the framework of Section 2. There we focused on the modifications in modern general equilibrium theory that would prevent individuals from carrying out all their trading plans at the initial date so that markets would reopen over time. Clearly, z^* is a trading arrangement in which markets will not reopen. Of course, trade occurs throughout $t \in T$, but these trades represent the execution of clauses of the grand contract made at the initial date.

A cause for markets reopening was the hypothesis that forward trading was more costly than spot. In the chicken–egg model the assumption of private information establishes this hypothesis by making the kind of forward trading described by z^* "impossible", or at least not strategically viable.

Sequential trading brings with it a new problem: it is necessary to keep track of each individual's trading history to enforce budget constraints. In the model of Section 2 this could be done in a rather simple way. Because it was common knowledge that the time horizon was fixed and finite, it sufficed to impose a single lifetime constraint that the sum of the values of all net trades must equal zero by the end of the trading horizon. Such a constraint is not possible in the chicken–egg model. [Furthermore, in a finite horizon version of the model, where it would be possible, it is not desirable; see Townsend (1982).]

The budget constraint underlying the optimal contract z must persuade each individual at each date that no matter what his previous history, the rewards from truthful reporting outweigh those from misrepresentation. This is achieved by keeping track of the individual's previous net contributions and allowing the individual to consume an annuitized amount based on this quantity. Green exhibits the similarities between the consumption function underlying the contract z and the permanent income hypothesis of Friedman (1957). Foley and Hellwig (1975) also draw this parallel in their study of the behavior of a single individual facing a similar problem.

Taking the chicken–egg model as representative of the budget enforcement problems with sequential trading, there is a clear-cut difference between such models and ones for which there is a single lifetime constraint. To help clarify the difference, let us distinguish between *prospective* wealth and *transactions* or recorded wealth. Prospective wealth is the expected discounted value of one's future endowment stream evaluated according to expected prices. Transactions wealth is the cumulated value of one's previous net trades.

In the forward/contingent contracts version of the chicken–egg model with a single lifetime budget constraint, transactions wealth is *not* a determinant of the optimal contract, z^*. However, in the private information version in which the enforcement problem is necessarily an integral feature of the trading arrangement, transactions wealth *is* a determinant of the optimal z. Indeed, one way to highlight the transactions role of money in comparison to the emphasis on its store of value function in the Walras–Hicks–Patinkin tradition is to call attention to the relative weights assigned to transactions and prospective wealth as determinants of behavior. In the Walras–Hicks–Patinkin tradition, the weight assigned to transactions wealth is nil.

5. The cash-in-advance constraint

Among recent approaches in monetary theory, the cash-in-advance model has been virtually the only one to focus on the transactions role of money. In this section we discuss two general equilibrium extensions of the cash-in-advance idea due to Grandmont and Younes (1972) and Lucas (1980) and applications of this model of the transactions role of money to changes in the money supply.

Compared to the complete symmetry with which all commodities enter the budget constraint in standard theory, the cash-in-advance condition singles out the money commodity by stipulating that sales of it *and no other commodity* can be used to finance current purchases. The background conditions underlying this restriction are that the timing of purchases and sales do not coincide and money obtained from current sales is not available until the next "period". On the face of it, the constraint would seem to be a rough approximation to a money economy without credit markets. However, once credit markets are introduced one could simply borrow money against current sales, for example through the use of credit cards, to relax the budget constraint to the point where it was, except for a small interest charge, of the usual form in which current sales of *all* commodities can be used to finance current purchases. In other words, with credit markets the standard form of the budget constraint would rather closely approximate the description of the choice set for an individual in a money economy.

Kohn, a forceful advocate of the cash-in-advance constraint [Kohn (1984)], has addressed this criticism in Kohn (1981). Acknowledging its validity as applied to the individual, he points out the fallacy of composition when applied to the economy as a whole. Since borrowing and lending net to zero, the money borrowed by one person to evade the cash constraint represents a corresponding tightening of the constraint for the lender. As a description of the exchange opportunities of the representative trader, the cash-in-advance constraint is applicable to an economy with credit markets. Similarly, objections to the highly stylized timing of payments in the model are also shown by Kohn to survive modifications. Modifications in the payments arrangement, however, cannot, and should not, be too drastic. There does have to be a limit of the speed of transactions. "If a model is to express money's role as medium of exchange, it cannot allow expenditure to be financed by contemporaneous income" [Kohn (1981, p. 192)].

5.1. Existence and quantity-theoretic properties of equilibrium

Grandmont and Younes (1972) responded to Hahn's critique of Patinkin using the suggestion proposed by Clower (see Subsection 1.2). Individuals have the lifetime intertemporal utility functions for the consumption stream $c = (c_0, c_1, \ldots, c_t, \ldots)$ used in Subsections 4.1 and 4.2. Each individual receives an endowment of perishable commodities w in each period so that an individual's lifetime endowment is

$$\omega = (w, w, \ldots) .$$

There are no forward markets but individuals do have expectations at each t about all prices $(p_{t+1}, p_{t+2}, \ldots)$ in the future which depend on prices at t and prices in the previous k periods. An individual's price expectations at t are given by the single-valued mapping $\psi(p_t, \ldots, p_{t-k}) = (p_{t+1}, p_{t+2}, \ldots)$. Price expectations are stationary in the sense that ψ does not depend explicitly on t. In addition, if prices have been constant in the past, it is assumed that they are expected to be constant in the future, $\psi(p, \ldots, p) = (p, p, \ldots)$.

The central feature of the model is the budget constraint. Letting c_t and m_{t+1} be the final allocations of goods and money while w and m_t are the initial allocations of goods and money at the start of period t, we have the accounting identity,

$$m_{t+1} = m_t + p_t \cdot (c_t - w) ,$$

as well as the modified cash-in-advance constraint,

$$p_t[c_t - w]^+ \leq m_t + kp_t \cdot [c_t - w]^- ,$$

where $k \in [0, 1]$ measures the "viscosity" of the payments arrangement. At one extreme is Clower's description of the cash-in-advance constraint where $k = 0$, and at the other is the traditional budget constraint where $k = 1$. In between, some but not all of current sales may be used to finance current purchases.

Standard quantity-theoretic propositions familiar from the Walras–Hicks–Patinkin tradition also apply here. For example, let $x_t = (c_t, m_t) = \xi(p, \ldots, p, m_{t-1})$ be the utility-maximizing demands of an individual when money balances available for purchases at t are m_{t-1} and prices at t and for the previous k periods have been equal to p. A *stationary state* satisfies the additional condition $m_t = m_{t-1}$, denoted by $x = (c, m) = \xi^*(p)$. Then $(c, \alpha m) = \xi^*(\alpha p)$, all $\alpha > 0$. Thus a doubling of prices and money balances leaves equilibrium real quantities c unchanged in long-run stationary equilibrium.

On the existence of a specifically monetary equilibrium where the individuals wish to hold the stock of money on hand, the authors show that there will exist such a stationary long-run equilibrium provided that at the equilibrium prices there is some trade, that $k < 1$, and that traders do not discount the future too much (β is not too small). The first two qualifications are precisely those necessary to give money a transactions function to perform. The third is required to give individuals sufficient concern for the future so that they care about making further trades.

The *classical dichotomy* between the monetary and the real sectors of the economy asserts that one can in effect solve for the equilibrium in a monetary economy first by finding equilibrium relative prices in the "barter" version of the model and then determining nominal prices by appealing to the quantity of money. The barter version of the model is, of course, the frictionless ideal. In such a scenario money is clearly an inessential veil covering the real allocation. The classical dichotomy is closely related to the optimal money supply mentioned in Subsection 4.1 and elsewhere in this Handbook.

When money is *not* needed for transactions, i.e. $k = 1$, the stationary long-run equilibrium is certainly Pareto optimal, and when $k < 1$, the stationary long-run equilibrium will typically be Pareto inferior, i.e. Pareto inferior to some other reallocation of resources that obeys only the overall aggregate constraints on resources but not necessarily any monetary exchange constraints. These remarks suffice to show that the classical dichotomy is not valid in the cash-in-advance model when money has a transactions role; or, the transactions role of money imposes a binding constraint on the allocation of resources compared to the frictionless barter ideal. (Compare Section 4 above.) However, if individuals discount the future very little, their willingness to hold money to finance future purchases increases and in the limit as the discount becomes nil (β approaches unity) Grandmont and Younes show that the stationary long-run equilibria of the model converges to an optimal barter allocation. In this limiting case the classical dichotomy does hold. [See Grandmont and Younes (1973) for other means to achieve this same conclusion.]

5.2. A cash-in-advance version of the chicken–egg model

In Subsection 4.2 we presented Green's version of the chicken–egg model, the primary purpose of which was to emphasize the similarity between the budget enforcement problems leading to monetary exchange and the necessary properties of incentive-compatible intertemporal trade when there is private information. In this subsection we describe Lucas's (1980) version of the chicken–egg model. The evident similarities to be brought out between the two should make the Lucas version with its institutionally imposed cash-in-advance constraint appear to be less arbitrary than it might otherwise seem.

Suppose, instead of random variations in endowments, each person's chicken always lays one egg each day but a person's current tastes for eggs are random. For example, on any day t one could have, with equal probability, a relatively strong desire ($s_t = 1$) or weak desire ($s_t = 0$) for eggs and these random variations are independent and identically distributed throughout $t \in T$. Letting $c = (c_1, c_2, \ldots, c_t, \ldots)$ be a sequence of quantities of eggs consumed in each period, where c_t may depend on the realization of $s^t = (s_0, \ldots, s_t)$, the expected discounted utility of c is given by

$$EU(c) = \sum_{t \in T} \beta^t E[u_{s_t}(c_t(s^t), s_t)] .$$

Assuming that random variations in tastes are common knowledge, we could create forward/contingent markets. With a continuum of individuals, the equilibrium trading arrangement might be as follows: when $s_t = 1$ set $c_t = 3/2$ and when $s_t = 0$ set $c_t = 1/2$. In comparison to the previous version of the chicken–egg model when the gains from trade came from the smoothing of consumption over time, here the gains come from being able to vary consumption with random changes in one's tastes. Note, however, that in terms of net trades, Green's version of the idea forward/contingent contracts model and Lucas's would be identical.

Lucas did not ask what the optimal incentive compatible intertemporal contract is. Instead, he proposed an institutional arrangement for trade that clearly is incentive compatible. Recall the distinction between prospective wealth and transactions wealth made in Subsection 4.2. Lucas's trading arrangement resolves the enforcement problem by adopting a budget constraint that effectively eliminates prospective wealth in favor of a constraint based entirely on transactions wealth.

Starting with a given non-negative amount of money, the individual is proscribed in his purchases precisely by a cash-in-advance constraint. The difference between one's money balances at date t and date $t+k$ represents the sum of the values of net trades between those two dates, or the change in the individual's transactions wealth. Of course, an individual is permitted to be a reckless spender if he has the money balances; however, since money holdings are non-negative and current purchases are constrained by current money holdings, the future costs of current spending can be perceived today.

Suppose initial money balances at t to be m_t and prices to be always unity. An individual receiving an endowment of one unit of a perishable commodity each period who has received the taste shock s_t will divide his transactions wealth and current income, m_t and 1, respectively, into current consumption c_t and money with which to start the next period m_{t+1} so as to solve the following:

$$V(m_t, s_t) = \max_{c_t, m_{t+1} \geq 0} \left\{ u(c_t, s_t) + \beta \int V(m_{t+1}, s_{t+1}) \, \mathrm{d}F(s_{t+1}) \right\}$$

subject to

$$m_{t+1} = m_t + 1 - c_t$$

and the cash-in-advance constraint

$$c_t \leq m_t \, .$$

Here V is the value function for the intertemporal maximization problem and F is the cumulative distribution function on the taste shock in each period. Since the optimal solution does not depend explicitly on t, we may describe the optimal choices by the policy functions $c_t = c(m_t, s_t)$ and $m_t = m(m_t, s_t)$.

The main contribution of Lucas's paper is the demonstration of a stationary stochastic equilibrium, i.e. a distribution of the population based on their money balances that is invariant from period to period. This equilibrium parallels the optimal contract in Green. Superficially, the "mechanism" associated with this equilibrium seems to be decentralized, whereas Green's mechanism does not. The latter specifies current consumption for the individual for every sequence of reported endowment histories, whereas the former allows the individual to make his own decisions as a function of his current money balances and taste shock. However, once the maximizing operation is performed, the Lucas equilibrium becomes a mechanism in which current consumption is a function of the history of past taste shocks (including the current one).

The recursive structure of c_t and m_t, above, imply that $c_t = m(m_t, s_t)$ can be written as $F_t(s_0, s_1, \ldots, s_t; m_0)$. Thus, a mechanism description of a net trade in the Lucus scheme is $z_t'(s_0, s_1, \ldots, s_t; m_0) = F_t - 1$. The difference between the two mechanisms is not so much decentralization as it is decentralization via prices. Lucas's scheme z' may not have the optimality properties of Green's optimal enforcement mechanism z, but it does represent an enforcement mechanism permitting decentralized decision-making through prices.

5.3. *Responses to changes in the money supply*

Consider the model of Subsection 5.1 in stationary long-run equilibrium. Suppose there is a one-time change in the money supply. With expectations function ξ arbitrarily given, it is difficult to say what the short-run response to this change will be. However, if expectations are "rational" and distributional effects can be ignored, there is an obvious prediction: the change in the money supply will not only have no long-run real consequences, it will also have no short-run consequences as well. For example, if the money supply doubles, and everyone now holds double their previous amounts of money, and the expectation is that current and future prices will double, there is an equilibrium in which the real allocation remains unchanged. The work discussed below aims to show that modifications in the cash-in-advance model do lead to short-run real consequences despite the imposition of correct expectations.

Distributional effects would cause a monetary injection to disturb prices in the short-run unless the injection were neutralized by distributing it on a pro-rata basis. Grossman and Weiss (1983) and Rotemberg (1984) move away from the "helicopter drops" method in exploring the short-run consequences of the distributional impacts of monetary injections. The monetary payments scheme itself is the source of the distributional impacts. They hypothesize a staggered payments scheme in which individuals hold money balances to make purchases for two periods at a time, rather than one. Inventory arguments of the Baumol–Tobin type are invoked to rationalize the advantages of making withdrawals only every other period. Also, to create a circular flow of money among households, half the population withdraws money for purchases at each date.

Grossman and Weiss analyze the consequences of an open-market purchase of bonds for money, say at date 1. That half of the population away from the bank at date 1, call them the B's, receives none of the extra money. The increase in money balances will have to be held by the half of the population that is at the bank, the A's. For the A's to hold the whole of the increased supply, their share of nominal spending must rise, e.g. if in a steady state A's share of spending was one-half, it will have to be greater than one-half in the first period after the open-market operation. But this will occur only if nominal and real interest rates decline. In a particular representation of this idea in a model with fixed output, they show that open market operations also lead to delayed price increases. In the long run the distribution effects work themselves out, but in the meantime there are systematic disturbances.

Rotemberg also developed a similar staggered system of money withdrawals with variable capital and output possibilities. Analyzing the effects of a one-time purchase of capital for money by the government, the distributional consequences of the monetary scheme leads not only to price increases but also to short-run increases in capital and output before returning to the steady-state equilibrium.

6. Concluding remarks

The transactions role of money is one of the most palpable of economic phenomena and would therefore seem to offer one of the first challenges for the theory of exchange. We know, of course, that the theory of exchange developed as a response to other challenges, notably as a theory of relative price determination and allocation of resources, topics for which it was appropriate to abstract from the frictions required to make room for money. In addition, the frictions required for monetary exchange, such as differential costs of spot and forward transactions and the strategic issues of incomplete information and incentive compatibility, are recent developments. This may explain the slow growth of the transactions role of money as a branch of the theory of exchange.

By now, however, the branch has clearly emerged and judged by the quantity of recent research it is undergoing a growth spurt. In this survey we have attempted to describe *some* of the contributions that have made it more visible. In this section, we provide a synopsis of key points.

The transactions role of money challenges the implicit logistical and informational assumptions of the theory of exchange. To begin, it is vital that trade be sequential, which involves more than the time-indexing of commodities. There are various sources of "sequentiality". One is the costs of making forward contracts in an otherwise highly organized market setting, which creates a need for markets to reopen over time. Another is the simple fact that in most instances individuals trade with each other one at a time.

The sequential nature of trade makes informational demands that go beyond the knowledge of prices that suffices in the traditional theory of exchange.

These informational elements underlie the disadvantages of barter. Most importantly, private information about one's own situation – present in unorganized as well as organized markets – introduces strategic problems of budget enforcement. There are many ways to cope with this problem. Whether through the comparatively primitive use of commodity money or the more sophisticated electronic funds transfer, money is a device to record and make public one's trading history.

A useful analogy can be made between the role of money as a record-keeping device and the theory of signalling [Spence (1974)]. Both originate from moral hazard. In signalling, divided knowledge makes it difficult to identify differences in economically relevant characteristics such as worker productivity. Compared to education which may be an observable proxy for productivity, money is quite a noisy signal. It says nothing about the personal characteristics of the bearer or the previous transactions that caused the individual to have these "credits". And, of course, it is this property of money that it only signals the lowest common denominator of personal characteristics that makes it a transferable signalling device.

Is the transactions role of money important? One way of measuring its importance is by the consequences of the budget enforcement problem for the allocation of resources. The natural yardstick for measurement is the departure from the no-enforcement-problem Walrasian ideal. A model in which the budget enforcement problem can be completely resolved through the (costless!) introduction of money so as to duplicate the allocation of resources in frictionless barter world can be regarded as a modern interpretation of the classical dichotomy. In such an economy, money, once present, is of no further consequence. The models of Section 2 and Subsection 3.2 fit this description.

While it seems reasonable and even desirable to examine the rationale behind the transactions role of money in a model exhibiting this dichotomy, if that were all there was, this would place a rather definite upper bound on the importance of the transactions role. Such models would serve as solutions to the intellectual puzzle: "Why money?" But there is no reason to believe that this dichotomy is valid, either in theory or in practice. Budget enforcement problems do have real consequences as the models of Sections 4 and 5 illustrate. There is even reason to believe that exploring the properties of models in which budget enforcement problems are always a binding constraint on behavior may illuminate our understanding of macroeconomics.

7. Bibliographic note

Section 2: Classical economists recognized money's role as an intertemporal asset. Formalization of the model with a sequence of budget constraints is, however, relatively recent. The fully explicit non-monetary general equilibrium model developed by Arrow and Debreu is fully expounded in Debreu (1959). The fully detailed sequence economy model with transaction costs, Hahn (1971), was presented as the Walras–Bowley lecture at the North American meeting of the Econometric Society in New York, 1969.

An antecedent of this model, without explicit monetary structure, was Radner (1972). Foley (1970) developed independently the transaction cost structure without the temporal sequence of markets. Work on efficiency includes Hahn (1973) and, most importantly, Starrett (1973), where the role of money and examples of inefficient equilibrium allocation in its absence are developed. Additional work on existence of equilibrium is due to Kurz (1974a, 1974b) and Heller (1974) and, in the case of non-convex transaction costs, to Heller and Starr (1976). The importance of the sequential structure of budgets is noted in exposition in Hahn (1982) and Gale (1982).

Alternative models emphasizing the reopening of markets include the overlapping generations model (treated elsewhere in this Handbook) and the temporary equilibrium model. The latter is discussed in Hicks (1939) and Arrow and Hahn (1971). The role of money as a portfolio asset there is due to Grandmont (1974), and Grandmont and Younes (1972, 1973); the field is surveyed in Grandmont (1977). Also notable in this regard are Bewley (1980, 1983), and Foley and Hellwig (1975).

Section 3: The classical economists, Smith (1775), Jevons (1893), and Menger (1892), clearly recognized the coordination problem in pairwise trade and money's function in alleviating it. Modern formal studies along this line include Niehans (1969, 1971), Ostroy (1973), Ostroy and Starr (1974), Starr (1972, 1976, 1986), Sontheimer (1970), Veendorp (1970), Eckalbar (1984, 1986), Norman (1987), Madden (1975, 1976), Graham, Jennergen, Peterson and Weintraub (1976), Feldman (1973), and Goldman and Starr (1982). Shubik (1973) is explicit that money facilitates coordination of separate, but necessarily interdependent, trading decisions.

Further contributions to the "Menger problem" are found in Kiyotaki and Wright (1989) and Iwai (1988). See O'Driscoll (1986) for the history.

King and Plosser (1986) develop a model of the informational advantages of money based on their physical properties. Modern antecedents of this tradition are Brunner and Meltzer (1971) and Alchian (1977).

Section 4: Gale (1980, 1982) and Townsend (1983, 1987) are two contributors who have emphasized the incentive issues underlying monetary exchange.

Section 5: An incomplete list of other work on the cash-in-advance approach is: Clower and Howitt (1978), Fried (1973), Jovanic (1982), Lucas and Stokey (1987), Stockman (1981), and Svensson (1985). Lucas (1988) contains a further study of the short-run consequences of changes in the money supply. Akerlof (1979, 1982) and Akerlof and Milbourne (1980) also study the short-run consequences of money shocks, but through an inventory policy framework rather than a cash-in-advance model. Scheinkman and Weiss (1986) exhibit the consequences for cyclical behavior when borrowing constraints are binding.

References

Akerlof, G.A. (1979) 'Irving Fisher on his head: The consequence of constant threshold-target monitoring of money holdings', *Quarterly Journal of Economics*, 93: 169–187.

Akerlof, G.A. (1982) 'The short-run demand for money: A new look at an old problem', *American Economic Review*, 72: 35–39.

Akerlof, G.A. and R.D. Milbourne (1980) 'Irving Fisher on his head II: The consequence of the timing of payments for the demand for money', *Quarterly Journal of Economics*, 94: 145–157.

Alchian, A. (1977) 'Why money?', *Journal of Money, Credit and Banking*, 9: 131–140.

Arrow, K.J. (1964) 'The role of securities in the optimal allocation of risk-bearing', *Review of Economic Studies*, 31: 91–96.

Arrow, K.J. and F.H. Hahn (1971) *General competitive analysis*. San Francisco: Holden-Day.

Baumol, W.J. (1952) 'The transactions demand for cash: An inventory theoretic approach', *Quarterly Journal of Economics* 66: 545–556.

Benveniste, L.J. (1987) 'Incomplete market participation and the optimal exchange of credit', in: E.C. Prescott and N. Wallace, eds., *Contractual arrangements for intertemporal trade*. Minneapolis: University of Minnesota Press.

Bewley, T. (1980) 'The optimum quantity of money', in: J.H. Kareken and N. Wallace, eds., *Models of monetary economies*. Minneapolis: Federal Reserve Bank of Minneapolis.

Bewley, T. (1983) 'A difficulty with the optimum quantity of money', *Econometrica*, 51: 1485–1504.

Brunner, K. and A. Meltzer (1971) 'The use of money: Money in the theory of an exchange economy', *American Economic Review*, 61: 784–805.

Clower, R.W. (1967) 'A reconsideration of the microfoundations of monetary theory', *Western Economic Journal*, 6: 1–8.

Clower, R. and P. Howitt (1978) 'The transactions theory of the demand for money: A reconsideration', *Journal of Political Economy*, 86: 449–466.

Debreu, G. (1959) *Theory of value*. New York: Wiley.

Eckalbar, J.C. (1984) 'Money, barter, and convergence to the competitive allocation: Menger's problem', *Journal of Economic Theory*, 32: 201–211.

Eckalbar, J.C. (1986) 'Bilateral trade in a monetized pure exchange economy', *Economic Modelling*, 3: 135–139.

Feenstra, R.C. (1986) 'Functional equivalence between liquidity costs and the utility of money', *Journal of Monetary Economics*, 17: 271–291.

Feldman, A.M. (1973) 'Bilateral trading processes, pairwise optimality and Pareto optimality', *Review of Economic Studies*, 40(4): 463–474.

Foley, D.J. (1970) 'Equilibrium with costly marketing', *Journal of Economic Theory*, 2: 276–291.

Foley, D.K. and M. Hellwig (1975) 'Asset management with trading uncertainty', *Review of Economic Studies*, 42: 327–346.

Fried, J. (1973) 'Money, exchange and growth', *Western Economic Journal*, 11: 285–301.

Friedman, M. (1957) *A theory of the consumption function*. New York: National Bureau of Economic Research.

Gale, D. (1978) 'The core of a monetary economy without trust', *Journal of Economic Theory*, 18: 456–491.

Gale, D. (1980) 'Money, information and equilibrium in large economies', *Journal of Economic Theory*, 23: 28–65.

Gale, D. (1982) *Money: In equilibrium*. New York: Cambridge University Press.

Goldman, S.M. and R.M. Starr (1982) 'Pairwise, t-wise, and Pareto optimalities', *Econometrica*, 50: 593–606.

Graham, D.A., L.P. Jennergen, D.W. Peterson and E.R. Weintraub (1976) 'Trader–commodity parity theorems', *Journal of Economic Theory*, 12: 443–454.

Grandmont, J.M. (1974) 'On the short run equilibrium in a monetary economy', in: J.H. Dreze, ed., *Allocation under certainty: Equilibrium and optimality*. U.K.: Macmillan; New York: Halsted Press–John Wiley & Sons, 213–228.

Grandmont, J.M. (1977) 'Temporary general equilibrium theory', *Econometrica*, 45: 535–572.

Grandmont, J.M. and Y. Younes (1972) 'On the role of the money and the existence of a monetary equilibrium', *Review of Economic Studies*, 39: 355–372.

Grandmont, J.M. and Y. Younes (1973) 'On the efficiency of a monetary equilibrium', *Review of Economic Studies*, 40: 149–165.

Green, E. (1987) 'Lending and the smoothing of uninsurable income', in E.C. Prescott and N. Wallace, eds., *Contractual arangements for intertemporal trade*. Minneapolis: University of Minnesota Press.

Grossman, S. and L. Weiss (1983) 'A transactions-based model of the monetary transmission mechanism', *American Economic Review*, 73: 871–880.

Hahn, F.H. (1965) 'On some problems of proving the existence of an equilibrium in a monetary economy', in: F.H. Hahn and F.P.R. Brechling, eds., *The Theory of Interest Rates*. London: Macmillan, 126–135.

Hahn, F.H. (1971) 'Equilibrium with transaction costs', *Econometrica*, 39: 417–439.

Hahn, F.H. (1973) 'On transaction costs, inessential sequence economies and money', *Review of Economic Studies*, 40: 449–461.

Harris, M. (1979) 'Expectations and money in a dynamic exchange model', *Econometrica*, 47: 1403–1419.

Heller, W.P. (1974) 'The holding of money balances in general equilibrium', *Journal of Economic Theory*, 7: 93–108.

Heller, W.P. and R.M. Starr (1976) 'Equilibrium with non-convex transactions costs: Monetary and non-monetary economies', *Review of Economic Studies*, 42(2): 195–215.

Hicks, J.R. (1935) A suggestion for simplifying the theory of money', *Economica* II, No. 5: 1–19. Reprinted in: J.R. Hicks, *Critical Essays in monetary theory*. Oxford: Oxford University Press, 1967.

Hicks, J.R. (1939) *Value and capital*. Oxford: Oxford University Press.

Iwai, K. (1988) 'The evolution of money – A search-theoretic foundation of monetary economics', CARESS Working Paper 88-03, University of Pennsylvania.

Jevons, W.S. (1893) *Money and mechanism of exchange*. New York: D. Appleton.

Jones, R.A. (1976) 'The origin and development of media of exchange', *Journal of Political Economy*, 84: 757–775.

Jovanovic, B. (1982) 'Inflation and welfare in the steady state', *Journal of Political Economy*, 90: 561–577.

Kareken, J.H. and N. Wallace, eds. (1980) *Models of monetary economies*. Minneapolis: Federal Reserve Bank of Minneapolis.

King, R.G. and C. Plosser (1986) 'Money as the mechanism of exchange', *Journal of Monetary Economics*, 17: 93–115.

Kiyotaki, N. and R. Wright (1989) 'On money as a medium of exchange', *Journal of Political Economy*, 97: 927–954.

Kohn, M. (1981) 'In defense of the finance constraint', *Economic Inquiry*, 19: 177–195.

Kohn, M. (1984) 'The finance (cash-in-advance) constraint come of age: A survey of some recent developments in the theory of money', Working Paper Series, Dartmouth College.

Kurz, M. (1974a) 'Equilibrium with transaction cost and money in a single market exchange economy', *Journal of Economic Theory*, 7: 418–452.

Kurz, M. (1974b) 'Equilibrium in a finite sequence of markets with transactions cost', *Econometrica*, 42: 1–20.

Lucas, R.E., Jr. (1980) 'Equilibrium in a pure currency economy', in: J.H. Kareken and N. Wallace, eds., *Models of monetary economies*. Minneapolis: Federal Reserve Bank of Minneapolis.

Lucas, R.E. (1988) 'Liquidity and interest rates', Unpublished manuscript, University of Chicago.

Lucas, R.E. and N. Stokey (1987) 'Money and interest in a cash-in-advance economy', *Econometrica*, 55: 491-513.
Madden, P.J. (1975) 'Efficient sequences of non-monetary exchange', *Review of Economic Studies*, 42: 581-596.
Madden, P.J. (1976) 'Theorem on decentralized exchange', *Econometrica*, 44: 787-791.
Menger, K. (1892) 'On the origin of money', *Economic Journal*, 2: 239-255.
Niehans, J. (1969) 'Money in a static theory of optimal payment arrangements', *Journal of Money, Credit and Banking*, 1: 706-726.
Niehans, J. (1971) 'Money and barter in general equilibrium with transactions costs', *American Economic Review*, 61: 773-783.
Norman, A.L. (1987) A theory of monetary exchange', *Review of Economic Studies*, 54: 499-517.
O'Driscoll, G.P. (1986) 'Money – Menger evolutionary – theory', *History of Political Economy*, 18: 601-616.
Oh, S. (1989) 'A theory of a generally acceptable medium of exchange and barter', *Journal of Monetary Economics*, 23: 101-119.
Ostroy, J.M. (1973) 'The informational efficiency of monetary exchange', *American Economic Review*, 63: 597-610.
Ostroy, J.M. and R.M. Starr (1974) 'Money and the decentralization of exchange', *Econometrica*, 42: 1093-1113.
Patinkin, D. (1965) *Money, interest and prices*. New York: Harper and Row.
Radner, R. (1968) 'Competitive equilibrium under uncertainty', *Econometrica*, 36: 31-58.
Radner, R. (1972) 'Existence of equilibrium of plans, prices and price expectations in a sequence of markets', *Econometrica*, 40: 279-296.
Rotemberg, J. (1984) 'A monetary equilibrium model with transactions costs', *Journal of Political Economy*, 92: 40-58.
Scheinkman, J.A. and L. Weiss (1986) 'Borrowing constraints and aggregate economic activity', *Econometrica*, 54: 23-45.
Shubik, M. (1973) 'Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model', *Western Economic Journal*, 4: 24-38.
Smith, A. (1775) *An inquiry into the nature and causes of the wealth of Nations*. Modern Library, New York: Random House.
Sontheimer, K. (1972) 'On the determination of money prices', *Journal of Money, Credit and Banking*, 4: 489-508.
Spence, M. (1974) *Market signaling: Information transfer in hiring and related processes*. Cambridge, Mass.: Harvard University Press.
Starr, R.M. (1972) 'The structure of exchange in barter and monetary economies', *Quarterly Journal of Economics*, 86: 290-302.
Starr, R.M. (1974) 'The price of money in a pure exchange monetary economy with taxation', *Econometrica*, 42: 45-54.
Starr, R. M. (1976) 'Decentralized non-monetary trade', *Econometrica*, 44: 1087-1089.
Starr, R.M. (1986) 'Decentralized trade in a credit economy', in: W.P. Heller, R. Starr and D. Starrett, eds., *Equilibrium analysis: Essays in honor of Kenneth J. Arrow*, vol. II. New York: Cambridge University Press.
Starr, R.M. ed. (1989) *General equilibrium models of monetary economies: Studies in the static foundations of monetary theory*. San Diego: Academic Press.
Starrett, D. (1973) 'Inefficiency and the demand for money in a sequence economy', *Review of Economic Studies*, 40: 289-303.
Stockman, A. (1981) 'Anticipated inflation and the capital stock in a cash-in-advance economy', *Journal of Monetary Economics*, 8: 387-393.
Svensson, L.E. (1985) 'Money and asset prices in a cash-in advance economy', *Journal of Political Economy*, 93: 919-944.
Taub, B. (1988) 'Efficiency in a pure credit economy', Virginia Tech. Working Paper.
Tobin, J. (1956) 'The interest-elasticity of transactions demand for cash', *Review of Economics and Statistics*, 38: 241-247.
Townsend, R.M. (1980) 'Models of money with spatially separated agents', in: J.H. Kareken and N. Wallace, eds., *Models of monetary economies*. Minneapolis: Federal Reserve Bank of Minneapolis.

Townsend, R.M. (1982) 'Optimal multiperiod contracts and the gain from enduring relationships under private information', *Journal of Political Economy*, 90: 1166–1186.
Townsend, R.M. (1983) 'Financial structure and economic activity', *American Economic Review*, 73: 895–911.
Townsend, R.M. (1987) 'Economic organization with limited communication', *American Economic Review*, 77: 954–971.
Townsend, R.M. and N. Wallace (1987) 'Circulating private debt: An example with a coordination problem', in: E.C. Prescott and N. Wallace, eds., *Contractual arrangements for intertemporal trade*. Minneapolis: University of Minnesota Press.
Ulph, A.M. and D.T. Ulph (1975) 'Transaction costs in general equilibrium theory – a survey', *Economica*, 42: 355–372.
Veendorp, E.C.H. (1970) "General equilibrium theory for a barter economy', *Western Economic Journal*, 8: 1–23.
Walras L. (1900) *Elements of pure economics*. Translated and edited by W. Jaffe. Homewood, Illinois: Irwin.