

The Price of Money in a Pure Exchange Monetary Economy with Taxation

Author(s): Ross M. Starr

Source: *Econometrica*, Vol. 42, No. 1 (Jan., 1974), pp. 45-54

Published by: [The Econometric Society](#)

Stable URL: <http://www.jstor.org/stable/1913684>

Accessed: 19/07/2011 15:29

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=econosoc>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



The Econometric Society is collaborating with JSTOR to digitize, preserve and extend access to *Econometrica*.

THE PRICE OF MONEY IN A PURE EXCHANGE MONETARY ECONOMY WITH TAXATION

BY ROSS M. STARR¹

Market determination of the value in exchange (price) of money is considered in a general equilibrium finite horizon model. The possibility of the price of money being zero in equilibrium and the role of taxes (payable in money) in preventing a zero price are considered.

1. THE ROLE OF MONEY AND ITS VALUE IN EXCHANGE

MONEY IS PECULIAR among commodities in that its usefulness depends on its price. It would not upset the theory of value if water or diamonds had a price of zero, but monetary theory depends on money having a positive value in exchange. The term "price of money" means here for money precisely what the "price" of any other commodity means for that commodity. Price is a real number which, taken in ratio with another such number, indicates a rate of exchange between two commodities. If $p^m > 0$ is the price of money and p^n is the price of good n , then p^n/p^m is the number of units of money which must be traded (spent) on the market in order to acquire (buy) one unit of good n . This usage is at variance with two standard practices: (i) taking money as numeraire, setting its "price" identically equal to unity, and (ii) referring to the interest rate as the "price of money." Neither of these two usages enters here. Unfortunately, it is far from clear that the equilibrium price will be positive [3, 7, 9, and 10]. This follows, after all, since modern money (debt instruments rather than items decorative or useful in themselves) generally consists of useless pieces of paper or accounting units whose only use is to be exchanged eventually for some positive quantity of other goods. However, if the price of money were zero, then for even arbitrarily large amounts of money one could buy precisely nothing. If we say that money is accepted because it is accepted, then we must agree that if money were not accepted then it would not be accepted because it would not be accepted. When the price of money is zero there will be no unsatisfied demand for money; there is an equilibrium in which the price of money is zero.

It is distressingly easy to find economies in which zero is the only price of money consistent with equilibrium. Consider an economy over time with a finite horizon. Near the terminal period the economy will be imbued with a *Weltuntergangstimmung*. There is no point in having a positive money holding at the end of the last period; at any positive price of money, money holders will seek to trade money for goods to be consumed before the end of the world.

¹ It is a pleasure to acknowledge the advice and criticism of K. J. Arrow. He bears no responsibility for errors in this essay. The research described in this paper was carried out under grants from the National Science Foundation and the Ford Foundation.

However, no one with any sense will accept money during the last period in exchange for goods. You can't take it with you. Money in the last period is useless, so the price of money in the last period will be zero. But then in the next to last period traders should be wary of accepting money. Since the price of money is zero in the last period there is no point in getting stuck with any money at the end of the next to last period, so the price of money will be zero in the next to last period as well. This argument can regress indefinitely so that the price of money is zero in all periods. Thus, in a discrete time finite horizon model in equilibrium the price of money will be zero in all periods. This is the argument of [9].

Though finite horizons are convenient to work with, we do not really believe in the end of the world occurring at a definite future date. Thus it is not too unreasonable to impose terminal conditions on money holdings—vaguely analogous to terminal capital stock constraints in finite horizon growth models—to eliminate depletion of money balances in the terminal period. Unfortunately there still may be an equilibrium where the price of money is zero.

How can we eliminate the possibility of the price of money being zero in equilibrium? In order to do this we must arrange that there be a positive excess demand for money when the price of money is zero. One way to achieve this is to guarantee that money can always be used in payment of taxes; that is, “the note debt of the state stands against a corresponding quantity of demands by the state which can be unconditionally satisfied by the notes” [5].

The modern state can make anything it chooses generally acceptable as money and thus establish its value quite apart from any connection, even of the most formal kind, with gold or with backing of any kind. It is true that a simple declaration that such and such is money will not do, even if backed by the most convincing constitutional evidence of the state's absolute sovereignty. But if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done. Everyone who has obligations to the state will be willing to accept the pieces of paper with which he can settle the obligations, and all other people will be willing to accept these pieces of paper because they know that the taxpayers, etc., will be willing to accept them in turn [6].

Taxes can be used to create a demand for money independent of its usefulness as a medium of exchange, thereby ensuring that its price will not fall to zero.

2. TRADE IN A MONETARY ECONOMY

There are N real goods; money is the $N + 1$ st good. The real goods will be denoted $n = 1, 2, \dots, N$. The $N + 1$ st good, money, is denoted m . Traders are elements of the set T . Each $t \in T$ has an endowment $\bar{x}^t \gg 0$, and a continuous utility function $u_t(x_t)$ on possible consumptions (elements of the nonnegative orthant of R^N).² We assume u_t is semi-strictly quasi-concave and fulfills strong monotonicity. That is, $x^1 > x^2$ implies $u_t(x^1) > u_t(x^2)$, and $\{y | u_t(y) \geq u_t(x)\}$ is convex for all x . Further, one distinguishes between buying, β , and selling, α , transactions. An individual's trades are characterized by what goods he buys and what goods he sells. Trader t 's trade will be represented by $y^t \in E^{2(N+1)+1}$; y^t has an entry $y^{t\delta n}$ for each of the

² Adopt the following convention on vector inequalities: $x \geq y$ means $x^i \geq y^i$ all i ; $x > y$ means $x^i \geq y^i$ all i with $x^i > y^i$ some i ; $x \gg y$ means $x^i > y^i$ all i .

$N + 1$ commodities n , and for each of the two possibilities, $\delta = \alpha, \beta$, selling or buying. The remaining co-ordinate of y^t represents the payment of taxes, $y^{t\tau}$. Trader t 's selling transactions are represented by $y^{t\alpha}$, which is composed of flows of goods from t to the market, and flows of money from the market to t :

$$(1) \quad y^{t\alpha n} \geq 0 \quad (n = 1, \dots, N),$$

and

$$(1m) \quad y^{t\alpha m} \leq 0.$$

Trader t 's buying transactions are represented by $y^{t\beta}$, which is composed of flows of goods from the market to t and flows of money from t to the market:

$$(2) \quad y^{t\beta n} \geq 0, \quad \text{all } n = 1, \dots, N,$$

and

$$(2m) \quad y^{t\beta m} \leq 0.$$

The vector of t 's buying transactions, selling transactions, and tax payments is y^t :

$$y^t = (y^{t\alpha}, y^{t\beta}, y^{t\tau}).$$

At the end of trade, trader t 's holdings will be subject to nonnegativity constraints. Trader t cannot sell what he did not have to start with and what he did not acquire in trade:

$$(3) \quad \bar{x}^{tn} + y^{t\beta n} - y^{t\alpha n} \geq 0, \quad \text{for } n = 1, \dots, N.$$

Price vectors will be elements of P , the unit simplex in E^{N+1} . Let $P = \{p | p \in E^{N+1}, p > 0, \sum_{n=1}^{N+1} p^n = 1\}$. Prices are the same for buying and selling. If one wished to extend the analysis to a model with transactions costs it should not be difficult to let buying and selling prices differ merely by doubling the dimensionality of the price space.

Taxes are paid in money and are distinct from other money expenditures only in that they do not buy anything for the trader. The nonnegativity constraint (3) should also be extended to money. Thus,

$$(4) \quad \bar{x}^{tm} - y^{t\alpha m} + y^{t\beta m} - y^{t\tau} \geq 0.$$

Further, introduce trader t 's required tax payment function (which may vary with prices) $\theta_t(p)$; then another constraint on trade is that traders pay their taxes. That is,

$$(5) \quad y^{t\tau} = \theta_t(p).$$

Trader t 's possible trades will be the set,

$$Y_t = \{y | y \in E^{2N+3}, y \text{ fulfills (1), (1m), (2), (2m), (3), (4), and (5)}\}.$$

Budget constraints apply separately to buying and selling transactions. In the standard general equilibrium model, of course, each trader faces only one budget constraint. The twofold budget constraint here reflects the two requirements

that the trader must supply to the market commodities equal in value to his money receipts from the market at market prices, and that the trader must pay to the market money equal in value at market prices to the goods he receives from the market. The constraints then are

$$(6\alpha) \quad p \cdot y^{t\alpha} = 0,$$

and

$$(6\beta) \quad p \cdot y^{t\beta} = 0.$$

Given prices, we can now write t 's trading opportunity set. This set consists of those trades consistent with payment of taxes, the budget constraints at prevailing prices, and the other requirements above ((1), (1m), (2), (2m), (3), and (4)). Thus, t 's trading opportunity set is

$$\eta_t(p) = \{y | y \in Y_t, y \text{ fulfills } (6\alpha) \text{ and } (6\beta) \text{ at } p\}.$$

It is from $\eta_t(p)$ that trader t will choose what trade to make. If he chooses $y \in \eta_t(p)$, then his consumption bundle consists of his original endowment plus his net trade. Consumption is $w^t = \bar{x}^t - y^\alpha + y^\beta$. The trader gets no satisfaction from money; utility varies only with the first N elements of w^t . Then t 's choice correspondence is $\gamma_t(p) = \{y | y \in \eta_t(p), w^t = \bar{x}^t - y^\alpha + y^\beta \text{ maximizes } u_t(w) \text{ subject to } y \in \eta_t(p)\}$.

It is fairly easy to see that the tax function, $\theta_t(p)$, if not suitably restricted, can make the analysis vacuous. For example, if $\theta_t(p)$ required trader t to make a large tax payment at unfavorable commodity prices—in particular if the value of the required tax payment were greater than the market value of the trader's endowment—then it might be impossible to satisfy simultaneously (1)–(6). In such a case $\eta_t(p)$ would be empty. To avoid this the following restriction is adopted:

RESTRICTION ON $\theta_t(p)$: For all $p \in P$,

$$p^m \theta_t(p) < p \cdot \bar{x}^t.$$

Also, to assure tractability assume strict positivity of endowment.

ASSUMPTION: $\bar{x}^t \gg 0$ for all $t \in T$.

The restriction says that no trader will be required to make a tax payment greater in value than his original endowment.

A recurrent problem in this family of models is that budget sets may fail to be continuous about $p^m = 0$. Consider $p^0 \in P$ with $p^{0m} = 0$, and for some commodities $l, n, p^{0l} > 0, p^{0n} = kp^{0n}, k > 0$. Then at prices p^0 a typical trader can buy none of good l . However, consider $p^v > p^0$ such that $p^{vm} > 0$ and $p^{vl} = kp^{vn}$. For any v , if $\theta_t(p^v)$ is not too big, there is $y^v \in \eta_t(p^v)$ such that $y^{vbl} = (1/k)\bar{x}^{tn} > 0$. But $y^0 \in \eta_t(p^0)$ implies $y^{0bl} = 0$. Hence $\eta_t(p)$ is not upper semi-continuous about $p^m = 0$. If we artificially bound $\eta_t(p)$, upper semi-continuity may be restored but failures of lower semi-continuity arise. Hahn noted this problem in a slightly different

context in [3]. It is also analogous to aspects of [4], where he found that budget sets may fail to be lower semi-continuous in some areas of the price space. The implication of this observation is that proofs of existence of equilibrium will not be able to rely on the continuity of demand functions, hence ruling out the direct application of fixed point theorems.

There is a formal identity between the model with taxation described above and a model lacking taxation but having some constraint on the depletion of the trader's money balance. Constraints (4) and (5) could be interpreted as a requirement that after completing trade the trader should have a money balance of at least $\theta_t(p)$. Special cases of this interpretation are $\theta_t(p) \equiv 0$ (nonnegativity of final balance) and $\theta_t(p) \equiv \bar{x}^{tm}$ (unchanged money holding). There are corresponding interpretations of the results below, relating the equilibrium price of money to the required final balance.

3. EQUILIBRIUM IN THE MONETARY ECONOMY

Let

$$\bar{\Omega} = X_{j \in T} Y_j.$$

On the basis of attempted trades we can compute excess demands. Let

$$\zeta(x) = \sum_{j \in T} x^{j\beta} - \sum_{j \in T} x^{j\alpha}, \quad \text{for } x \in \bar{\Omega}.$$

Thus for a proposed group of transactions $x^j, j \in T$, the excess demand $\zeta(x)$, is the amount sought for purchase, $\sum_{j \in T} x^{j\beta}$, less the amount traders seek to supply, $\sum_{j \in T} x^{j\alpha}$.

DEFINITION: Let $p^* \in P$, $y^* \in \bar{\Omega}$, $z^* \in R^N$. Then (p^*, y^*, z^*) is an equilibrium for the economy if for all $t \in T$ (i) $y^{*t} \in \eta_t(p^*)$, for all $t \in T$, (ii) $w^t = \bar{x}^t + y^{*t\beta} - y^{*t\alpha}$ maximizes $u_t(w)$ for all $y \in \eta_t(p)$, and (iii) $z^* = \zeta(y^*)$ and $z^* \leq 0$.

This is a traditional definition of equilibrium. One has an equilibrium when the results of individual maximizations subject to constraint imply non-positive excess demand. The novel elements are embodied in the constraints that transactions take place through money, (6. α) and (6. β), and that taxes be paid in full, in money, (4) and (5).

4. THE PRICE OF MONEY

We now note a curious property of the monetary economy. When the value in exchange of money is zero, no one can seek to trade. We have required that all trade take place using money as medium of exchange. If the price of money is zero, then money is literally worthless paper. What will one sell for worthless paper? Nothing. What can one buy with worthless paper? Nothing. Thus we have the following lemma.

LEMMA 1: Let $p \in P$, $p^m = 0$. Then $y \in \eta_j(p)$ implies $p^n y^{\delta n} = 0$, $\delta = \alpha, \beta$ for all $j \in T$, $n = 1, \dots, N$. The same holds for all $y \in \gamma_j(p)$.

PROOF: $\gamma_j(p) \subset \eta_j(p)$. However, $y \in \eta_j(p)$ implies y fulfills (6.α) and (6.β) at p . Thus $p \cdot y^\alpha = 0$ and $p \cdot y^\beta = 0$. Let $y^{\delta c}$ denote the N -dimensional vector consisting of the first N components (the real goods elements) of y^δ , $\delta = \alpha, \beta$. Then

$$p \cdot y^\delta = 0, \quad \delta = \alpha, \beta,$$

and

$$p \cdot y^\delta = p^c \cdot y^{\delta c} + p^m \cdot y^{\delta m}, \quad p > 0.$$

By (1), (1m), (2), and (2m) we have $y^{\delta m} \leq 0$, $y^{\delta c} \geq 0$. However $p^m = 0$ implies $p^m \cdot y^{\delta m} = 0$,

$$0 = p \cdot y^\delta = p^c \cdot y^{\delta c} + p^m \cdot y^{\delta m} = p^c \cdot y^{\delta c}.$$

Thus $p^n \cdot y^{\delta n} = 0$, $n = 1, \dots, N$.

Q.E.D.

Denote the first $2(N + 1)$ components of y , those dealing with market transactions, by y^π ; the final component, that dealing with tax payments, is, of course, y^τ .

LEMMA 2: Let $p \in P$; $p^m = 0$, $p^n > 0$, all $n = 1, \dots, N$. Then for each $j \in T$, $y \in \eta_j(p)$ implies $y^c = 0$. If $\theta_j(p) \leq \bar{x}^{jm}$, then there is $y^0 \in \eta_j(p)$ and $\gamma_j(p)$ with $y^{0\pi} = 0$.

PROOF: By Lemma 1 we have $y \in \eta_j(p)$ implies $p^n y^{\delta n} = 0$. However $p^n > 0$ for $n = 1, \dots, N$, so $y^{\delta n} = 0$ for $n = 1, \dots, N$. Thus $y^c = 0$ for $\theta_j(p) \leq \bar{x}^{jm}$ implies we can satisfy (5) with $y^m = 0$; thus let $y^0 = (0, 0, \dots, 0, \theta_j(p))$; $y^0 \in \eta_j(p)$. Further, since u_j does not vary with w^{jm} , j is indifferent among all elements of $\eta_j(p)$. Thus $y^0 \in \gamma_j(p)$.

Q.E.D.

This leads us to the fundamental quandary of this study. Lemma 2 tells us that if we announce a price vector p for the market such that the price of money is zero, the price of goods is positive, and taxes are sufficiently small, then traders will demand and supply zero quantities of all goods. But if all individual demands and supplies are zero, then excess demands and supplies are zero in all markets. The markets are in equilibrium, and p is an equilibrium price vector. This is a very curious equilibrium, however, since it is an equilibrium with no trade. This is not to say that there are no mutually beneficial trades conceivable between traders. Rather, because it is required in the monetary economy that trade take place through money, there are no effective demands or supplies when the price of money is zero. I think there is a legitimate question as to the significance of the equilibrium with zero price of money. Within the bounds of the model the implication is explicit: no trade. An alternative interpretation, going outside the model, is that there is no monetary trade, but that there is probably recourse to barter (see [10]). The implication of the structure of our demand functions is the following theorem.

THEOREM 1: Let $p^0 \in P$, $p^{0m} = 0$, and $\theta_j(p^0) < \bar{x}^{jm}$, all $j \in T$. Then there is an equilibrium (p^0, y^0, z^0) such that $y^{0n} = 0$.

PROOF: Let $p^{0m} = 0$ and $p^{0n} > 0$, $n = 1, \dots, N$. By Lemma 2, $y^{0j} \in \gamma_j(p^0)$ with $y^{0jn} = 0$. However $\zeta(0) = 0$, so $(p^0, y^0, 0)$ is an equilibrium. Q.E.D.

There is only one case in which the equilibrium $(p^0, y^0, 0)$ can be Pareto efficient. It will be Pareto efficient if and only if the original endowment of goods, \bar{x}^j , is a Pareto efficient allocation.

LEMMA 3: Let $p^m > 0$, $p^n > 0$ for some $n = 1, \dots, N$. Then let $\gamma \in \gamma_j(p)$. Then

$$\bar{x}^{jm} - y^{jm} + y^{\beta m} = \theta_j(p).$$

PROOF: Suppose (4) is overfulfilled by the amount a_j . Then there is $*y \in \eta_j(p)$ with $*y^{\alpha n} = y^{\alpha n}$, $*y^{\beta m} < y^{\beta m}$, and $*y^{\beta n} \geq y^{\beta n}$ with the strict inequality holding for some n . Further by strong monotonicity of u , $*y$ is preferred to y , so $y \notin \gamma_j(p)$ contrary to hypothesis. The contradiction proves the lemma. Q.E.D.

The gist of Lemma 3 is that when the price of money is positive the nonnegativity restriction (4) is binding. The mischief one gets into then when the tax functions θ_j are insufficiently exacting now arises. When the price of money is positive, traders will deplete their money holdings to the point where the nonnegativity constraint is binding. If the constraint is not restrictive enough, however, this will result in an excess supply of money on the market and an excess demand for goods, clearly a disequilibrium. If we do not make the tax functions more exacting, the sole alternative is to let the price of money become zero. This gives us an equilibrium of the sort described in Theorem 1.

THEOREM 2: Let $\sum_{j \in T} \theta_j(p) < \sum_{j \in T} \bar{x}^{jm}$ for all $p \in P$. Let (p, y, z) be an equilibrium for the economy. Then $p^m = 0$.

PROOF: By insatiability of u_j , $p^n > 0$, some n . Suppose the theorem is false; $p^m > 0$. Then by Lemma 3

$$\sum_{j \in T} (\bar{x}^{jm} - y^{j\alpha m} + y^{j\beta m}) < \sum_{j \in T} \bar{x}^{jm},$$

which implies $\sum_{j \in T} y^{j\alpha m} > \sum_{j \in T} y^{j\beta m}$. However, we have $0 = p \cdot y^{j\alpha} = p^0 \cdot y^{j\alpha c} + p^m \cdot y^{j\alpha m}$ and $0 = p \cdot y^{j\beta} = p^0 \cdot y^{j\beta c} + p^m \cdot y^{j\beta m}$. We have then

$$y^{j\beta m} = \frac{-p^c y^{j\beta c}}{p^m} \quad \text{and} \quad y^{j\alpha m} = \frac{-p^c y^{j\alpha c}}{p^m}.$$

By the inequality above, then,

$$\sum_{j \in T} \left(-\frac{p^c}{p^m} \right) \cdot y^{j\alpha c} > \sum_{j \in T} \left(-\frac{p^c}{p^m} \right) \cdot y^{j\beta c},$$

so for some $n = 1, \dots, N$, $\sum_{j \in T} y^{jan} < \sum_{j \in T} y^{j\beta n}$. However, $z^n = \sum_{j \in T} y^{j\beta n} - \sum_{j \in T} y^{jan} > 0$, and therefore (p, y, z) is not an equilibrium. The contradiction shows that $p^m = 0$. Q.E.D.

Theorem 2 tells us that not only do we face the difficulty of Theorem 1 (that there exist equilibria with price of money equal to zero) but also that in a broad class of cases (those where the tax functions are not exacting enough) the *only* equilibria are those where the price of money is zero. Such a situation could make life in a monetary economy awkward indeed. In this model the final demand for money is based on taxes. As illustrated in Theorem 2, when that constraint does not require sufficiently large terminal money holdings, the demand for money is not sufficiently great to lift the equilibrium price above zero. It will appear below that the converse holds, at least partially; when taxes are sufficiently high, but not so great as to be impossible, the only equilibria will be those with a positive price of money.

Thus, we can introduce the following theorem.

THEOREM 3: *Let $\theta_j(p)$ be $\theta_j(p^m)$, a function of p^m only. Let $\theta_j(p)$ be such that there is $0 \leq b < 1$ so that for all $p \in P$, $0 \leq p^m \leq b$ and $\sum_{j \in T} \theta_j(p) > \sum_{j \in T} \bar{x}^{jm}$. Then there is an equilibrium for the economy, and if (p^0, y^0, z^0) is such an equilibrium, then $p^{0m} > b$.*

PROOF: Note that the restriction on $\theta_j(p)$: (i) implies that if $p^m = 1$ and $p^c = 0$, then $\theta_j(p) < \bar{x}^{jm}$, and (ii) together with strict positivity of endowment implies that the requirement $\sum_{j \in T} \theta_j(p) > \sum_{j \in T} \bar{x}^{jm}$ and $p^m \theta_j(p) < p \cdot \bar{x}^j$, all $j \in T$ and all p so that $0 \leq p^m \leq b$, is not a contradiction. There exist such $\theta_j(p)$. Then by the intermediate value theorem there is p^{*m} , $b \leq p^{*m} < 1$, so that $\sum_{j \in T} \theta_j(p^{*m}) = \sum_{j \in T} \bar{x}^{jm}$. Consider the barter (Arrow-Debreu) economy with prices on the simplex,

$$S = \{p | p \in E^N, p \geq 0, \sum p^i = 1 - p^{*m}\}$$

where traders, $t \in T$, choose x^{tc} to maximize utility subject to the budget constraint

$$p \cdot x^{tc} \leq p \cdot \bar{x}^{tc} - (p^{*m} \theta_t(p^{*m}) - p^{*m} \bar{x}^{tm}).$$

Then there is an equilibrium price vector $p^{*c} \in S$ in this economy [2]. Let

$$y^{tax} = (\bar{x}^{tc} - x^{tc})^+,$$

$$y^{tam} = \frac{-1}{p^{*m}} p^{*c} \cdot y^{tax},$$

$$y^{t\beta c} = (\bar{x}^{tc} - x^{tc})^-,$$

and

$$y^{t\beta m} = -\frac{1}{p^{*m}} p^* \cdot y^{t\beta c}.$$

Then $p^* = (p^{*c}, p^{*m})$ is an equilibrium price vector for the economy according to the definition given in Section 3. This gives existence.

Suppose, contrary to the theorem, $p^{0m} \leq b$. Then we have

$$\bar{x}^{0jm} - y^{0jzm} + y^{0j\beta m} \geq \theta_j(p),$$

$$\sum_{j \in T} \bar{x}^{0jm} - \sum_{j \in T} y^{0jzm} + \sum_{j \in T} y^{0j\beta m} \geq \sum_{j \in T} \theta_j(p) > \sum_{j \in T} \bar{x}^{jm},$$

and

$$\sum_{j \in T} y^{0j\beta m} - \sum_{j \in T} y^{0jzm} > 0.$$

There is excess demand for m and hence (p^0, y^0, z^0) is not an equilibrium. The contradiction proves the theorem. Q.E.D.

Theorem 3 tells us that if we can get traders to fulfill the right sort of tax constraint, $\theta_j(p)$, the price of money will be positive.

In an equilibrium with a positive price of money, trade takes place unimpeded. Just as a competitive equilibrium is Pareto efficient in a barter economy [1], so a competitive equilibrium with a positive price of money is Pareto efficient.

THEOREM 4: *Let (p^0, y^0, z^0) be an equilibrium for the economy, and let $1 > p^{0m} > 0$. Then $w^{0t} = \bar{x}^t - y^{0t\alpha} + y^{0t\beta}$ is a Pareto efficient distribution of goods among $t \in T$.*

PROOF: By [1] it is sufficient to show that w^{0t} maximizes $u_t(x)$ subject to $p^{0c} \cdot w^{0tc} \geq p^{0c} \cdot x^c$. Suppose not; then there is $r \in T$, so that for some x^{rc}

$$p^{0c} \cdot x^{rc} \leq p^{0c} \cdot w^{0rc}, \quad u_r(x^{rc}) > u_r(w^{0r}).$$

We will show that this implies that y^{0r} is not a maximizing choice in $\eta_r(p^0)$ and hence is a contradiction of the hypothesis. Without loss of generality take $p^{0c} \cdot x^{rc} = p^{0c} \cdot w^{0rc}$. Choose y^r so that

$$y^{r\alpha c} = (x^{rc} - \bar{x}^{rc})^-,$$

$$y^{r\alpha m} = -p^{0c} \cdot (x^{rc} - \bar{x}^{rc})^- \frac{1}{p^{0m}},$$

$$y^{r\beta c} = (x^{rc} - \bar{x}^{rc})^+,$$

and

$$y^{r\beta m} = -p^{0c} \cdot (x^{rc} - \bar{x}^{rc})^+ \frac{1}{p^{0m}}.$$

Since $p^{0c} \cdot (x^{rc} - \bar{x}^{rc}) = p^{0c} \cdot x^{rc} - p^{0c} \bar{x}^{rc} = p^{0c} \cdot w^{0rc} - p^{0c} \cdot \bar{x}^{rc} = 0$, $y^r \in \eta_r(p^0)$. Thus (p^0, y^0, z^0) cannot be an equilibrium. The contradiction proves the theorem.

Q.E.D.

5. SUMMARY

In the pure exchange monetary economy, we have the following statements:

- (i) There exists an equilibrium (Theorems 1 and 3).
- (ii) If taxation is insufficiently exacting, there is an equilibrium with the price of money equal to zero (Theorem 1); further, this may be the only price of money consistent with equilibrium (Theorem 2).
- (iii) If taxes are sufficiently exacting, there are equilibria and they all have positive price of money (Theorem 3).
- (iv) An equilibrium with positive price of money is Pareto efficient (Theorem 4).

Cowles Foundation, Yale University

Manuscript received October, 1971; revision received February, 1972.

REFERENCES

- [1] ARROW, K. J.: "An Extension of the Basic Theorem of Classical Welfare Economics," in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, ed. J. Neyman. Berkeley: University of California Press, 1951.
- [2] DEBREU, GERARD: *Theory of Value*. New York: John Wiley, 1959.
- [3] HAHN, FRANK H.: "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy," in *Conference on the Theory of Interest and Money*, ed. F. P. R. Brechling. Royau-mont, France, 1962.
- [4] ———: "Equilibrium with Transactions Costs," *Econometrica*, 37 (1971), 417–439.
- [5] KAULLA, RUDOLPH: *Grundlagen des Geldwerts*. Stuttgart, 1920. Passage translated in *German Monetary Theory: 1905–1933*, Howard S. Ellis. Cambridge, Mass.: Harvard University Press, 1934.
- [6] LERNER, ABBA P.: "Money as a Creature of the State," *Proceedings of the American Economic Association*, 37 (1947), 312–317.
- [7] MARSCHAK, JACOB: "Money and the Theory of Assets," *Econometrica*, 6 (1938), 311–325.
- [8] ———: "The Rationale of the Demand for Money and of 'Money Illusion,'" *Metroeconomica*, (1950), 71–100.
- [9] PATINKIN, DON: "The Indeterminacy of Absolute Prices in Classical Economic Theory," *Econo-metrica*, 17 (1949), 1–27.
- [10] SONTHEIMER, KEVIN: "On the Determination of Money Prices," *Journal of Money Credit and Banking*, forthcoming.