MIDTERM EXAMINATION

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Ross or Jong — who promise to be clueless), until the examination is collected.

The exam is due at 1:30, in class, Wednesday, February 10, 2010.

Answer all 4 (four) questions. .

All notation not otherwise defined is taken from Starr's General Equilibrium Theory, *draft second edition. If you need to make additional assumptions to answer a question, that's OK. Do state the additional assumptions clearly.*

1. Let there be two commodities x,y in a pure exchange economy economy. The possible household consumption set is the nonnegative quadrant. $X^i \equiv \mathbf{R}^{\mathbf{2}}_{+}$. All households have the same preferences $\succeq_i$ characterized in the following way:

$(x^\circ, y^\circ) \succ_i (x', y')$ if $x^\circ + y^\circ > x' + y'$, OR if $x^\circ + y^\circ = x' + y'$ and $x^\circ > x'$.

$(x^\circ, y^\circ) \sim_i (x', y')$ if $(x^\circ, y^\circ) = (x', y')$.

(a) The preferences $\succeq_i$ do not fulfill C.V (Continuity) of Starr's *General Equilibrium Theory*. Give a mathematical demonstration of this property. A full proof is not required. What are the implications for demand behavior of the household?

(b) Assume the economy with household preferences $\succeq_i$ fulfills all of the assumptions of Starr's *General Equilibrium Theory*, draft second edition, Theorem 14.1, with the exception of C.V. In order to assure C.VII (Adequacy of Income), assume for all i, that $r^i \geq (2, 2)$ where the inequality holds co-ordinatewise.

In this economy, does there exist a competitive general equilibrium price vector? If 'yes' provide a demonstration or find an equilibrium price vector and equilibrium allocation (assume any additional properties of r^i needed to solve for the equilibrium allocation). If 'no' provide a demonstration. If the answer is 'possibly but not always,' explain fully. A full proof is not required.

2. On the island of Vinopesce there are two perfectly divisible products: wine, y, and fish, x. The only factor of production is perfectly divisible labor, L. Maximum fish catch for the whole island is 100 fish. This is a static equilibrium problem: there are no conservation issues. There are ten perfectly competitive fishing firms, denoted j = 1, 2, ..., 10. Labor employed by firm j is denoted by L^j and by firm i (typically a dummy index) is L^i. All firms have the same technology

$$x^j = L^j, \text{ when } \sum_{i=1}^{10} L^i < 100$$

$$x^j = 100\frac{\mathrm{L^j}}{\sum_{\mathrm{i=1}}^{10} \mathrm{L^i}}, \text{ when } \sum_{i=1}^{10} L^i \geq 100$$

Firms behave myopically with regard to the congestion effect in fishing. Firm j treats $\sum_{i=1}^{10} L^i$ parametrically as Q, assuming

$$x^j = 100\frac{\mathrm{L^j}}{\mathrm{Q}}, \text{ when } Q \geq 100$$

Q denotes the total labor employed in fishing, treated parametrically by all firms (that is, firms do not recognize their own contribution to Q in optimizing their response to the Lindahl pricing scheme). Note that x^j is not differentiable with respect to Q at $Q = 100$, but for all values of $Q > 100$ we have $\frac{\partial x^j}{\partial Q} = -\frac{100}{Q^2}L^j$. Thus for $Q > 100, Q \approx 100, \frac{\partial x^j}{\partial Q} \approx -\frac{1}{100}L^j$.

Wine is produced under constant returns by many firms k, with the technology, $y^k = L^k$. There are 1000 laborers on Vinopesce, one per household, each endowed with one unit of (divisible) labor. All households have the same utility function

$$u^h(x^h, y^h) = x^h + .5y^h, \text{ where } x^h \text{denotes h's fish consumption, and } y^h$$

denotes h's wine consumption. Leisure is not valued. Households sell their labor at the competitive wage rate w. Set $p_y = 1$. That is, wine is the numeraire with price unity. The following quantities are determined in competitive equilibrium:

$$w = \text{competitive wage rate of labor} = 1$$

$$\sum_{i=1}^{10} x^i = \text{total fish harvest} = 100$$

$$\sum_{i=1}^{10} L^i = \text{total labor employed in fishing} = 200$$

$$p_x = \text{price of fish} = 2$$

Consequent wine output is 800. This competitive allocation is inefficient because of the congestion externality in fishing. A superior attainable allocation is 100 fish with 100 laborers employed in fishing and 900 wine. A Lindahl auctioneer proposes the following Lindahl pricing scheme to treat the externality:

Q denotes the total labor employed in fishing, treated parametrically by all firms (that is, firms do not recognize their own contribution to Q in optimizing their response to the Lindahl pricing scheme). Each firm i, can specify a desired level of Q, Q^i, based on its own optimization and the Lindahl price of Q^i , t^i.

$$t^j = \text{ firm j's Lindahl price of } Q, \; t = \sum_{i=1}^{10} t^i$$

Firm j's Lindahl profit function is

$\pi^j = p_x x^j - wL^j + t^j Q^j - tL^j$ where x^j, L^j, Q^j are j's decision variables.

Q^j is firm j's selected level of Q given t^j. A Lindahl equilibrium occurs where there is market clearing in wine, fish, labor, and where $Q^j = Q = \sum_{i=1}^{10} L^i$ for all firms j. The Lindahl auctioneer recognizes that there is a variety of allocations of labor across firms consistent with a Lindahl equilibrium. He proposes $t^i = 0.1$, all i, as appropriate Lindahl pricing leading to an efficient allocation. The Lindahl auctioneer is thinking of an efficient allocation $L^i = 10$, all i.

Is there a Lindahl equilibrium at the value of t^i specified?
If not, explain why.
If so, find the allocation of labor across firms, and between wine and fish. Are firms optimizing their Lindahl profit functions? Is the allocation Pareto efficient? Explain fully, finding values for x^j, L^j, Q^j.

3. Starr's *General Equilibrium Theory*, draft second edition, problems 14.10, 14.11, 14.12.

4. Starr's *General Equilibrium Theory*, draft second edition, problem 15.1.