Psychology and Economics
(Lecture 17)

Xavier Gabaix

April 13, 2004

Vast body of experimental evidence, demonstrates that discount rates are higher in the short-run than in the long-run.

Consider a final thought experiment:

- Choose a ten minute break today or a fifteen minute break tomorrow.
- Choose a ten minute break in 100 days or a fifteen minute break in 101 days.

Resolution to discount rate evidence:

Adopt discount functions that imply high discount rates in the short-run and low discount rates in the long-run.

E.g., $\Delta(\tau) = \frac{1}{1+\kappa\tau}$ instead of $\Delta(\tau) = \delta^{\tau}$. If $\Delta(\tau) = \frac{1}{1+\kappa\tau}$, then the instantaneous discount rate will be:

$$
\begin{aligned}
-\frac{d\Delta(\tau)/d\tau}{\Delta(\tau)} &= -\frac{d\frac{1}{1+\kappa\tau}/d\tau}{\frac{1}{1+\kappa\tau}} \\
&= \frac{\kappa(1+\kappa\tau)^{-2}}{(1+\kappa\tau)^{-1}} \\
&= \frac{\kappa}{(1+\kappa\tau)}
\end{aligned}
$$

The discount rate declines with τ. Moreover, as τ goes to infinity, the discount rate goes to zero.

0.1 Quasi-hyperbolic discount functions:

- So far, we've worked with discount functions that are defined in both continuous-time

$$t \in [0, \infty)$$

and discrete-time

$$t \in \{0, 1, 2, ...\}.$$

- For example, δ^{τ} and $\frac{1}{1+\kappa\tau}$ are defined for all non-negative values of τ.

- We'll now consider a discount function that is only defined for discrete time.

- The *quasi-hyperbolic discount function* (HDF) is:

$$\Delta_H(\tau) = \begin{cases} 1 & \text{if } \tau = 0 \\ \beta \cdot \delta^\tau & \text{if } \tau \in \{1, 2, ...\} \end{cases}.$$

We'll sometimes use the easier notation:

$$\{\Delta_H(\tau)\}_{\tau=0}^{\infty} = \{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\}$$

- Note that this function looks a little bit like the exponential discount function:

$$\Delta_E(\tau) = \delta^\tau = \begin{cases} 1 & \text{if } \tau = 0 \\ \delta^\tau & \text{if } \tau \in \{1, 2, ...\} \end{cases}$$

Here again, it is more natural to write:

$$\{\Delta_E(\tau)\}_{\tau=0}^{\infty} = \{1, \delta, \delta^2, \delta^3, ...\}$$

Let's consider the HDF in greater detail.

- We'll typically assume that $\beta \simeq \frac{1}{2}$ and $\delta \simeq 1$

- For these values the HDF takes on values

$$\{\Delta_H(\tau)\}_{\tau=0}^{\infty} = \{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...\}$$

- Intuition: relative to the current period, all future periods are worth less (weight $\frac{1}{2}$).

- Most (for this example, *all*) of the discounting takes place between the current period and the immediate future.

- There is little (for this example, *no*) additional discounting between future periods.

- HDF captures the property that most discounting occurs in the short-run — utils today are twice as valuable as utils tomorrow.

- In the long-run we're relatively patient — utils tomorrow are just as valuable as utils the day after tomorrow.

Thinking about the discount rate in discrete time.

- We can't differentiate $\Delta(\tau)$ with respect to τ if this function is only defined at discrete points $\tau \in \{0, 1, 2, ...\}$

- So we use differences to approximate derivatives.

- In continuous-time we define the discount rate at horizon τ as the rate of decline of the discount function

$$-\frac{d\Delta(\tau)/d\tau}{\Delta(\tau)}$$

- In discrete-time we define the discount rate at horizon τ as the rate of decline of the discount function

$$-\frac{\Delta(\tau) - \Delta(\tau - 1)}{\Delta(\tau - 1)}$$

- For the exponential discount function, this discrete-time definition implies that the discount rate is

$$\begin{aligned} -\frac{\Delta_E(\tau) - \Delta_E(\tau - 1)}{\Delta_E(\tau - 1)} &= -\frac{\delta^\tau - \delta^{\tau-1}}{\delta^{\tau-1}} \\ &= 1 - \delta \\ &\simeq -\ln \delta \end{aligned}$$

- Note that this discrete-time discount rate does not depend on the horizon $\tau \in \{1, 2, 3, ...\}$..

- Note too that this discrete-time discount rate is approximately equal to the discount rate implied by the continuous-time definition.

- For HDF, discount rate depends on the horizon.

- When $\tau = 1$

$$\hat{\rho}(\tau) = -\frac{\Delta_H(1) - \Delta_H(0)}{\Delta_H(0)} = -\frac{\beta\delta - 1}{1}$$
$$= 1 - \beta\delta$$
$$(= \frac{1}{2})$$

- For our example, short-run discount rate is 50%.

- When $\tau \in \{2, 3, 4, ...\}$

$$\hat{\rho}\,(\tau) = -\frac{\Delta_H(\tau) - \Delta_H(\tau-1)}{\Delta_H(\tau-1)} = -\frac{\beta\delta^{\tau} - \beta\delta^{\tau-1}}{\beta\delta^{\tau-1}}$$
$$= 1 - \delta$$
$$\simeq -\ln\delta$$
$$(\ = 0)$$

- For our example, long-run discount rate is 0%.

0.2 Doing a problem set with a commitment technology

- Consider the decision of when to do a problem set.

- The cost of doing the problem set increases, the later that you start (say that starting it later makes it more difficult to get help from the TA).

- You could do the problem set during one of three periods.

- At date 0, the cost of doing the problem set is 1.

- At date 1, the cost of doing the problem set is $\frac{3}{2}$.

- At date 2, the cost of doing the problem set is $\frac{5}{2}$.

- You have a hyperbolic discount function:

$$\{\Delta(\tau)\}_{\tau=0}^{2} = \{1, \beta\delta, \beta\delta^2\}.$$

- To simplify matters, set $\beta = \frac{1}{2}$ and $\delta = 1$, so

$$\{\Delta(\tau)\}_{\tau=0}^{2} = \{1, \frac{1}{2}, \frac{1}{2}\}.$$

- Assume that the agent has a commitment technology (study group). When would the agent commit to do the problem set?

- If the problem set is done at period 0, the discounted cost is $\Delta(0) \cdot 1 = 1$.

- If the problem set is done at period 1, the discounted cost is $\Delta(1) \cdot \frac{3}{2} = \frac{3}{4}$.

- If the problem set is done at period 2, the discounted cost is $\Delta(2) \cdot \frac{5}{2} = \frac{5}{4}$.

- So commit to do the problem set at period 1.

0.3 Doing a problem set without commitment.

- Now suppose that the student does not have access to a commitment technology.

- Suppose the student is naive about her own behavior, and thinks that whatever plans she makes today will actually be carried out.

- This is the naive case.

- So at date 0, she wants to do the problem set at date 1. We proved this on the previous slide.

- So at date 0, she doesn't do the problem set expecting to do it at date 1.

- Assume that period 0 has passed and period 1 has arrived.

- We'll refer to the self that makes decisions at period 1 as "self 1."

- What does self 1 want to do?

- If the problem set is done "today" (period 1), the discounted cost from the perspective of self 1 is $\Delta(0) \cdot \frac{3}{2} = \frac{3}{2}$.

- If the problem set is done "tomorrow" (period 2), the discounted cost from the perspective of self 1 is $\Delta(1) \cdot \frac{5}{2} = \frac{5}{4}$.

- So self 1 does NOT want to do the problem set during period 1.

- Self 1 prefers to again postpone the problem set another period.

- Self 1 delays the problem set until period 2, violating the wishes of self 0.

- Is this procrastination? Self 0 planned to do the problem set in period 1.

- Self 1, delays the problem set until period 2.

Naives:

- Naive selfs choose under the (false) assumption that later selves will do what the earlier self wants.

- In the problem above, the naif equilibrium is to do the problem set in period 2, even though self 0 expects to do the problem set in period 1.

Sophisticates:

- Sophisticate selfs make decisions based on *correct* beliefs about the choices of later selves.

- The sophisticate equilibrium is the same as a subgame perfect Nash equilibrium. Every self has correct beliefs above the play of later selves, and every self optimizes in every subgame.

How would a sophisticate play the game above? Use backwards induction.

- By assumption, if the problem set is not done in either period 0 or 1, then the problem set will be done in period 2. (It has to get done, since period 2 is the last period in the game.)

- Now go back to period 1.

- Self 1 has the choice of doing the problem set in period 1 or leaving the problem set to period 2.

- Self 1 prefers to wait.

$$\Delta(0) \cdot \frac{3}{2} = \frac{3}{2}$$
$$\Delta(1) \cdot \frac{5}{2} = \frac{5}{4}$$

So if it's not done before period 1, it won't get done until period 2.

- Self 0 would like the task to be done in period 1.

$$\Delta(0) \cdot 1 = 1$$
$$\Delta(1) \cdot \frac{3}{2} = \frac{3}{4}$$
$$\Delta(2) \cdot \frac{5}{2} = \frac{5}{4}$$

But, self 0 correctly anticipates that self 1 won't do it after all.

- So self 0 faces the effective choice of doing it in period 0 or doing it in period 2. Period 0 is preferred ($1 < \frac{5}{4}$), so self 0 does it in period 0 to avoid delaying it until period 2.

In this problem sophisticates did the problem set “too early” to avoid doing it “too late.” However, sometimes even sophisticates can do things “too late.”

Consider the following slightly different cost structure:

- At date 0, the cost of doing the problem set is 1.
- At date 1, the cost of doing the problem set is $\frac{5}{6}$.
- At date 2, the cost of doing the problem set is $\frac{3}{2}$.

With commitment, it will be done in period 1.

Without commitment, naifs will do it in period 2.

Without commitment, sophisticates will do it in period 2!

0.4 What's wrong with these models?

0.4.1 Naives

- Consider a naif with $\beta = \frac{1}{2}$ and $\delta = 1$.

- The naif has to finish a project by deadline T.

- In time period t, the (undiscounted) project costs $\left(\frac{3}{2}\right)^t$ utils to execute.

- When will the naif do the project?

From the current self's perspective, it's always better to postpone doing the project until next period:

$$\begin{aligned}\left(\frac{3}{2}\right)^{t} &> \beta\delta\left(\frac{3}{2}\right)^{t+1}\\ &= \frac{1}{2}\left(\frac{3}{2}\right)^{t+1}\\ &= \frac{3}{4}\left(\frac{3}{2}\right)^{t}\end{aligned}$$

When will the project be completed?

0.4.2 Sophisticates:

Consider the same model as above.

When will a sophisticate do the project?

On the next problem set you will prove the following two claims:

1. If T is even, then sophisticates will do the project in even periods (and not in odd periods).

2. If T is odd, then sophisticates will do the project in odd periods (and not in even periods).