Economics 208 Problems **Winter 2004**

Vincent Crawford, Department of Economics, UCSD

1-2 (counts as two questions). Consider the Battle of the Sexes game. Assume, here and below, that the structure is common knowledge, that both players are self-interested, and that there are no observable differences between the players or their roles in the game. In each of the variations described below, say whether you would expect the players to be able to coordinate on one of the efficient pure-strategy equilibria, and what strategies you would expect the players to use, on average. Briefly but clearly explain your answers.

	Fights	**Ballet**
Fights	3, 1	0, 0
Ballet	0, 0	1, 3

Battle of the Sexes

(a) The original simultaneous-move game is a complete model of the players' situation.

(b) The game is modified so that Row chooses her/his strategy first and Column gets to observe her/his choice before choosing her/his own strategy.

(c) Row chooses her/his strategy first and Column does NOT get to observe her/his choice before choosing her/his own strategy.

(d) Row chooses her/his strategy first, Column observes her/his choice before choosing her/his own strategy, but Row then gets to observe Column's choice and costlessly revise her/his own choice, and this decision ends the game (so that Column cannot revise her/his choice).

(e) The original simultaneous-move game is a complete model of the players' situation, except that Row (only) can make a non-binding suggestion about the strategies players should use before they choose them.

(f) The original simultaneous-move game is a complete model of the players' situation, except that both players can make simultaneous, non-binding suggestions about the strategies players should use before they choose them.

(g) The original simultaneous-move game is a complete model of the players' situation, except that players can make sequential, non-binding suggestions about the strategies players should use before they choose them, say with Row making the first suggestion.

3. Consider the following two-person guessing game. Each player has her/his own target, lower limit, and upper limit. These are possibly different across players, and they influence players' payoffs as follows. Players make simultaneous guesses, which are required to be within their limits. Each player then earns 1000 points minus the distance between her/his guess and the product of her/his target times the other's guess.

(a-d) Find the Nash equilibrium or equilibria for the following targets and limits:

a)

	Lower Limit	Target	Upper Limit
Player 1	200	0.7	600
Player 2	400	1.5	700

b)

	Lower Limit	Target	Upper Limit
Player 1	300	0.7	500
Player 2	400	1.3	900

c)

	Lower Limit	Target	Upper Limit
Player 1	400	0.5	900
Player 2	300	0.7	900

d)

	Lower Limit	Target	Upper Limit
Player 1	300	1.3	500
Player 2	200	1.5	900

(e) State and prove a general result that determines the equilibrium as a function of the targets and limits for these guessing games.

(f) Would you expect intelligent people randomly paired from students who have not taken this class to play their equilibrium strategies in these guessing games? Explain why or why not. If not, explain what you think they might do instead.

4. Two risk-neutral, expected money-maximizing bargainers, U and V, must agree on how to share \$1. They bargain by making simultaneous demands; if their demands add up to more than \$1, they each get nothing; if they add up to less than or equal to \$1, each bargainer gets exactly his demand. Assume that any real number is a possible demand, and is also a possible division of the money.

(a) Find at least an infinite number of mixed-strategy Nash equilibria in this game. Explain why, in your equilibria, neither bargainer can do better by switching to any other strategy, pure or mixed.

(b) Show how to compute the equilibrium probability of disagreement, and show that it is always strictly positive in the mixed-strategy equilibria you identified in part (a).

(c) Are there any Pareto-efficient equilibria in this game?

(d) Now suppose that there are two plausible, but rival, notions of what it means to divide the dollar fairly. Redo your analysis from part (a), assuming that bargainers can put positive probability only on demands that are consistent with one or the other notion of fairness. Is the equilibrium identified here also an equilibrium in the original game?

(e) Give a fairly detailed real-world (but not experimental, even if you think experiments are "real") example in which common ideas of fairness appear to determine bargaining outcomes (and the likelihood of impasse) as in your answer to (d).

5. Consider a two-person game with payoff matrix as shown. Before choosing simultaneously between T and B, or L and R, Column must send R a costless, nonbinding ("cheap talk") message announcing her/his intention to play either L or R. Assume that both players know the rules of the game, including the values of *x* and y, as common knowledge.

	L	R
T	2, 2	0, x
B	1, 1	1, y

(a) For what values of x and y are the choices T for Row and L for Column (each with probability one) consistent with subgame-perfect equilibrium in the entire game?
(b) For what values of x and y are the choices T for Row and L for Column (each with probability one) each part of some rationalizable strategy (in the entire game)?

In "Nash Equilibria are not Self-Enforcing" (in *Economic Decision Making: Games, Econometrics and Optimisation*, edited by Gabszewicz, Richard, and Wolsey, Elsevier 1990) Aumann argues that in games like this with $x \geq y$, an announcement by Column that s/he intends to play L should not (or will not, in a positive theory) alter Row's belief that Column will actually play L, because Column does as well or better when Row plays T without regard to whether Column plays L or R.

(c) Do either subgame-perfect equilibrium or rationalizability distinguish between the credibility of an announcement by Column that s/he intends to play L when $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$?

(d) What assumptions about strategic behavior suffice to justify Aumann's argument against the credibility of such an announcement.

(e) Evaluate the credibility of an announcement by Column that s/he intends to play L behaviorally, making whatever assumptions and using whatever arguments and evidence you find useful. What, if any, meanings might such an announcement convey beyond those it conveys in arguments based on subgame-perfect equilibrium or rationalizability? Make clear how and why your evaluation of the credibility of the announcement distinguishes between games where $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$.

6. Suppose three identical, risk-neutral firms must decided simultaneously and irreversibly whether to enter a new market which can accommodate only two of them. If all three firms enter, all get payoff 0; otherwise, entrants get 9 and firms that stay out get 8.

(a) Identify the unique mixed-strategy equilibrium and describe the resulting probability distribution of the total ex post number of entrants. (You are not asked to show this, but the game also has three pure-strategy equilibria, in each of which exactly two firms enter; but these equilibria are arguably unattainable in a one-shot game in the absence of prior agreement or precedent. The mixed-strategy equilibrium is symmetric, hence attainable.)

Now suppose that each firm follows a behavioral rule that is an independent and identically distributed draw from a distribution that assigns equal probabilities to two types: either L1 (best response assuming the other firms are each equally likely to enter or stay out, and probabilistically independent), or L2 (best response to L1).

(b) Describe the decisions of types L1 and L2 and the resulting *actual* (as opposed to what L1 or L2 expect) probability distribution of the total ex post number of entrants when each firm's type is drawn as explained above. Show that the expected number of entrants is closer to the ex post optimal number (2) than in your equilibrium from part (a), and that that the probability of exactly 2 entrants is higher than in (a). (In experiments subjects' initial responses come systematically closer to ex post optimality than the symmetric mixed-strategy equilibrium predicts, a result Kahneman has described as "magic." This analysis shows that bounded strategic rationality works like fairy dust.)

Now suppose that each firm follows a rule that is an independent and identically distributed draw from a distribution that assigns probability ½ to type L1, ¼ to L2, and ¼ to a type called Sophisticated, which plays an equilibrium in the game in which the prior probabilities of L1, L2, and Sophisticated players are common knowledge.

(c) Plugging in the behaviors of L1 and L2 players (which do not depend on the prior type probabilities), characterize equilibrium in the game played by Sophisticated players.

(d) How does your answer to (c) change, if at all, if the prior probability of Sophisticated players is $\varepsilon \approx 0$, and the prior probability of L2 players is $½ - \varepsilon$ (with the prior probability of L1 players held constant at ½)?

7. (In memory of Bob Rosenthal; see his paper with Dale and Morgan, “Coordination through Reputations: A Laboratory Experiment,” *Games and Economic Behavior* 38 (2002), 52-88.) Suppose a large group of people are repeatedly, randomly, and anonymously paired to play the Hawk-Dove game below. The game is symmetric, there is nothing to distinguish player’s roles, and the people are indistinguishable, with one exception: each person’s past realized history of play (an ordered sequence of pure actions, such as H,H,D,D,D,H,D,…) is made public within each pair before they choose their actions in the current play. Imagine for simplicity that each person is randomly assigned an initial one-period history, either H or D, before play begins.

	Hawk	**Dove**
Hawk	0 0	1 2
Dove	2 1	0 0

(a) Describe informally but clearly at least three qualitatively different kinds of (pure or mixed) repeated-game strategies that are consistent with symmetric subgame-perfect equilibrium in the game played by the entire group, with the payoffs of their strategies evaluated before the uncertainty of pairing is resolved.

(b) Identify a symmetric subgame-perfect equilibrium that is as efficient as possible. (Note that the restriction to symmetric equilibria makes this equivalent to maximizing the expected lifetime payoff of a representative player.)

(c) How would expect this game to be played by intelligent, well-motivated, self-interested real people?

8. Write a one-page summary and critique of one of these papers from the reading list

Andrew Schotter, Keith Weigelt, and Charles Wilson, "A Laboratory Investigation of Multiperson Rationality and Presentation Effects," *Games and Economic Behavior* 6 (1994), 445-468

David Cooper and John Van Huyck, "Evidence on the Equivalence of the Strategic and Extensive Form Representation of Games," *Journal of Economic Theory* 110 (2003), 290-308

Richard McKelvey and Thomas Palfrey, "An Experimental Study of the Centipede Game," *Econometrica* 60 (1992), 803-836

Dale Stahl and Paul Wilson, "On Players' Models of Other Players: Theory and Experimental Evidence," *Games and Economic Behavior* 10, (1995), 218-254

Teck-Hua Ho, Colin Camerer, and Keith Weigelt, "Iterated Dominance and Iterated Best Response in Experimental '*p*-Beauty Contests'," *American Economic Review* 88 (1998), 947-969

Russell Cooper, Douglas DeJong, Robert Forsythe, and Thomas Ross, "Selection Criteria in Coordination Games: Some Experimental Results," *American Economic Review* 80 (1990), 218-233

Teck Hua Ho and Keith Weigelt, "Task Complexity, Equilibrium Selection, and Learning: An Experimental Study," *Management Science* 42 (1996), 659-679

Alvin Roth and Francoise Schoumaker, "Expectations and Reputations in Bargaining: An Experimental Study," *American Economic Review* (1983), 362-372

Lones Smith and Ennio Stachetti, "Aspirational Bargaining," manuscript, 2002 (http://www-personal.umich.edu/~lones/ftp/aspire.pdf)

Judith Mehta, Chris Starmer, and Robert Sugden, "The Nature of Salience: An Experimental Investigation of Pure Coordination Games," *American Economic Review* 84 (1994), 658-674

John Van Huyck, Raymond Battalio, and Frederick Rankin, "On the Origin of Convention: Evidence from Coordination Games," *Economic Journal* 107 (1997), 576-597

Jordi Brandts, and Charles Holt, "An Experimental Test of Equilibrium Dominance in Signaling Games," *American Economic Review* 82 (1992), 1350-1365

Jeffrey Banks, Colin Camerer, and David Porter, "An Experimental Analysis of Nash Refinements in Signaling Games," *Games and Economic Behavior* 6 (1994), 1-31

Ray Battalio, F. Rankin, and John Van Huyck, "Strategic Similarity and Emergent Conventions Evidence from Similar Stag Hunt Games," *Games and Economic Behavior*, 32 (2000), 315-337

David Cooper and John Kagel, "Learning and Transfer in Signaling Games," manuscript, 2002 (not online; pdf sent by request)

Ido Erev and Alvin E. Roth, "Predicting how people play games: Reinforcement Learning in Experimental Games with Unique, Mixed Strategy Equilibria," *American Economic Review* 88 (1998), 848-881