Fatal Attraction:

Focality, Naivete, and Sophistication in Experimental "Hide-and-Seek" Games

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Abstract: Rubinstein, Tversky, and Heller elicited subjects' initial responses to games in which a Hider and a Seeker choose simultaneously among four locations, with the Seeker winning a given amount if he chooses the same location as the Hider and the Hider otherwise winning that amount. The game has a unique equilibrium, in which both players randomize uniformly over locations; but the design framed locations non-neutrally and subjects deviated systematically from equilibrium in ways that were highly sensitive to the framing. This paper compares alternative explanations of the results and proposes a structural non-equilibrium model of initial responses to explain them.

Keywords: experimental game theory, framing effects, focal points, bounded rationality, strategic sophistication

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1. Introduction

Game theorists and economists have been intrigued by "Hide-and-Seek" games for more than 50 years (von Neumann (1953)). Rubinstein and Tversky (1993; henceforth "RT") and Rubinstein, Tversky, and Heller (1996; "RTH") were perhaps the first to study such games experimentally; see also Rubinstein (1999; "R"). RT, RTH, and R (collectively "RTH" from now on) elicited subjects' initial responses to several closely related Hide-and-Seek games. In a leading example of their games, one player, the Hider, hides a "treasure" or "prize" in one of four locations; and the other player, the Seeker, looks in one of the locations. Because the Seeker looks without observing the Hider's choice, their choices are strategically simultaneous. If the Seeker chooses the same location as the Hider he wins a given amount; if not, the Hider wins that amount.

Instead of giving subjects a payoff matrix, RTH explained the Hide-and-Seek games in "stories," probably increasing comprehension. Their key innovation was to present the games with non-neutral framing of the locations. R, for example, told Seekers: "You and another student are playing the following game: Your opponent has hidden a prize in one of four boxes arranged in a row. The boxes are marked as follows: A, B, A, A. Your goal is, of course, to find the prize. His goal is that you will not find it. You are allowed to open only one box. Which box are you going to open?" Hiders were told an analogous story. In effect the entire structure, including the order and labeling of locations, was publicly announced, with the goal of making it common knowledge.

This story makes the framing of locations non-neutral in two ways. The "B" location is uniquely distinguished by its label, and is thus focal in one of Schelling's (1960) senses. And the two "end A" locations, though not distinguished by their labels, may be inherently focal, as RT and RTH argue, citing Christenfeld (1995).[2] As RTH note, these two focalities interact to give the remaining location, "central A," its own brand of uniqueness as "the least salient location" (RT).

RTH's design is important as a tractable abstract model of applications in which games like Hide-and-Seek are played on naturally occurring, non-neutral, cultural or geographic "landscapes." Equilibrium analysis ignores such landscapes unless they directly influence the payoff structure. Consider, for instance, the complete-information Hide-and-Seek game derived from the above example, presented as a payoff matrix in Figure 1 with each player's winning payoff normalized to

[2]See also R's discussion of Ayton and Falk's (1995) two-dimensional Hide-and-Seek experiment.

1.[3] This game has a unique equilibrium, in which both players randomize uniformly, treating the B, central A, and end A locations symmetrically.[4] Because the Seeker can look in only one of the four locations, the Hider has an advantage, with equilibrium expected payoff ¾ versus ¼ for the Seeker.

Hider/Seeker	**A**	**B**	**A**	**A**
A	0, 1	1, 0	1, 0	1, 0
B	1, 0	0, 1	1, 0	1, 0
A	1, 0	1, 0	0, 1	1, 0
A	1, 0	1, 0	1, 0	0, 1

Figure 1. Hide-and-Seek Game

Despite these clear equilibrium predictions, the publicly announced order and labeling of locations create a potential for framing effects, and RTH's subjects deviated systematically from equilibrium in ways that were highly sensitive to framing.[5] Table I gives the aggregate choice frequencies for the "RTH-4" treatment described above and RTH's most closely related treatments. In RTH-4 the "least salient" central A location is the modal choice for both Hiders and Seekers, and is even more prevalent for Seekers than Hiders. As a result, the frequency with which Seekers find the prize exceeds ¼ and Seekers have higher expected payoffs than in equilibrium. These patterns extend, properly interpreted, to the other five treatments in the top half of Table I, which we argue in Section 2 are closely analogous to RTH-4. They also extend, with minor exceptions, to a large sample from more recent web experiments (Ariel Rubinstein, private communication); and with variations explained below to the less closely analogous treatments in the bottom half of Table I.

These systematic deviations from equilibrium are particularly intriguing because they are asymmetric across player roles, while equilibrium yields symmetric predictions. RT and RTH took them as prima facie evidence that their subjects did not think strategically, and with one exception noted in Section 2, they did not consider alternative explanations. In RT's words, "The finding that both choosers and guessers selected the least salient alternative suggests little or no strategic

[3]RT's and RTH's subjects' payments appeared sufficient to motivate them, and the binary-lottery structure of the payoff function implies under standard assumptions that players maximize expected money payoffs, without regard to risk preferences. Note however that in R's experiments, subjects were not paid or screened for exposure to game theory.
[4]Von Neumann (1953) characterized equilibria for such games allowing payoffs to vary across locations; and for a two-dimensional game in which the Hider hides in a matrix and the Seeker tries to guess either the Hider's Row or Column.
[5]Although in this game any strategy, pure or mixed, is a best response to equilibrium beliefs, systematic deviations of aggregate choice frequencies from equilibrium probabilities are symptomatic of deviations from equilibrium beliefs. Other studies of framing effects in games include Scharlemann et al. (2001), who studied trust games in which otherwise anonymous players were "labeled" by photographs; and Mehta, Starmer, and Sugden (1994), who studied coordination games in which decisions had naturally occurring labels as in Schelling's (1960) classic experiments.

thinking." In RTH's words, "In the competitive games, however, the players employed a naïve strategy (avoiding the endpoints), that is not guided by valid strategic reasoning. In particular, the hiders in this experiment either did not expect that the seekers too, will tend to avoid the endpoints, or else did not appreciate the strategic consequences of this expectation."

Table I. Aggregate Choice Frequencies in RTH's Experiments

RTH-4	**A**	***B***	**A**	**A**
Hider (53)	9%	36%	40%	15%
Seeker (62)	13%	31%	45%	11%
RT-AABA-Treasure	**A**	**A**	***B***	**A**
Hider (189)	22%	35%	19%	25%
Seeker (85)	13%	51%	21%	15%
RT-AABA-Mine	**A**	**A**	***B***	**A**
Hider (132)	24%	39%	18%	18%
Seeker (73)	29%	36%	14%	22%
RT-1234-Treasure	**1**	***2***	**3**	**4**
Hider (187)	25%	22%	36%	18%
Seeker (84)	20%	18%	48%	14%
RT-1234-Mine	**1**	***2***	**3**	**4**
Hider (133)	18%	20%	44%	17%
Seeker (72)	19%	25%	36%	19%
R-ABAA	**A**	***B***	**A**	**A**
Hider (50)	16%	18%	44%	22%
Seeker (64)	16%	19%	54%	11%
RTH-1	***Triangle***	**Circle**	**Circle**	**Circle**
Hider (53)	23%	23%	43%	11%
Seeker (62)	29%	24%	42%	5%
RTH-2	**Polite**	***Rude***	**Honest**	**Friendly**
Hider (53)	15%	26%	51%	8%
Seeker (62)	8%	40%	40%	11%
RTH-3	**Smile**	**Smile**	**Smile**	***Frown***
Hider (53)	21%	26%	34%	19%
Seeker (62)	7%	25%	34%	34%
RTH-5	**Frown**	***Smile***	**Frown**	**Frown**
Hider (53)	15%	40%	34%	11%
Seeker (62)	16%	55%	21%	8%
RTH-6	**Hate**	**Detest**	***Love***	**Dislike**
Hider (53)	11%	23%	38%	28%
Seeker (62)	20%	21%	55%	14%

Sample sizes in parentheses; focal labels in italics; order of presentation of locations to subjects as shown.

In our view, however, such robust patterns are unlikely to lack a coherent explanation; and given the simplicity of the strategic questions Hide-and-Seek games pose, the explanation is unlikely to be nonstrategic. On the contrary, such systematic deviations from equilibrium in games where its rationale is especially strong seem a particularly promising "proving ground" for alternative, non-equilibrium theories of strategic behavior. Understanding RTH's results is likely to prove helpful in many applications involving games like Hide-and-Seek played on non-neutral landscapes, where people's choices are sensitive to the landscape but equilibrium ignores it.

This paper compares alternative explanations of RTH's results and conducts an illustrative econometric analysis that helps to discriminate among them. We focus on explaining the patterns of deviation from equilibrium that are common to RTH-4 and, mutatis mutandis, the five most closely analogous RTH treatments in the top half of Table I: the prevalence of central A (or its analog, as explained in Section 2) for both Hiders and Seekers, its greater prevalence for Seekers (or their analogs), and the fact that Seekers (or their analogs) find the treasure more often than in equilibrium. We seek a parsimonious explanation that rests on behaviorally plausible assumptions.

Section 2 explains the other treatments in Table I and the senses in which they are analogous to RTH-4, and reports tests for differences in choice frequencies across treatments.

Section 3 considers what may be the simplest way to try to explain RTH's results: an equilibrium analysis of the Hide-and-Seek game with payoff perturbations that reflect "hard-wired" preferences about the salient focally labeled and/or end locations. This model can explain the prevalence of central A for both Hiders and Seekers by postulating behaviorally plausible perturbations of equal magnitudes but opposite signs across player roles, with Hiders averse to focally labeled and/or end locations and Seekers favoring them (Figure 2). But the model can only explain the greater prevalence of central A for Seekers by allowing large, unexplained differences across roles in the perturbations' magnitudes as well as their signs. Such differences give the model enough flexibility to explain almost any pattern, raising concerns about overfitting.

Section 4 considers explanations based on a generalization of equilibrium called Quantal Response Equilibrium ("QRE"; McKelvey and Palfrey (1995)), which explains the patterns of deviations from equilibrium in some other experiments. In a QRE players' choices are noisy, with the probability of each choice increasing in its expected payoff, given the distribution of others' choices; a QRE is thus a fixed point in the space of players' choice distributions. The specification is completed by a response distribution, whose noisiness is represented (inversely) by a precision

parameter. Some of our results are independent of this distribution, but for others we adopt the standard assumption of logit responses and study the special case called "logit QRE".

Hide-and-Seek without payoff perturbations has enough symmetry to make QRE coincide with equilibrium for any error distribution, but perturbations that are sensitive to framing give QRE the potential to explain RTH's results. Logit QRE can explain the prevalence of central A for Hiders and Seekers with perturbations of equal magnitudes but opposite signs across player roles, as in Section 3's perturbed equilibrium model. However, the main difficulty is explaining the greater prevalence of central A for Seekers, and logit QRE with perturbations of equal magnitudes but opposite signs robustly predicts that central A is more prevalent for *Hiders*.[6] Like equilibrium, logit QRE can only explain RTH's results by postulating large differences across roles in the perturbations' magnitudes as well as their signs. With or without such differences, however, the maximum likelihood estimate of the precision parameter is effectively infinite, reducing the logit QRE model to the analogous equilibrium with perturbations model from Section 3.[7]

These analyses show that equilibrium or QRE with payoff perturbations can only explain RTH's results by postulating essentially unrestricted differences in players' payoffs across roles. Besides creating a risk of overfitting, such differences beg the question of why the Hide-and-Seek game elicits responses that are systematically asymmetric across player roles. Accordingly, Section 5 returns to Hide-and-Seek without payoff perturbations and considers explanations based on a structural non-equilibrium model that explains the patterns of deviation from equilibrium in some other experiments that elicit initial responses to games (Stahl and Wilson (1994, 1995); Nagel (1995); Ho, Camerer, and Weigelt (1998); Costa-Gomes, Crawford, and Broseta (2001); Camerer, Ho, and Chong (2004); and Costa-Gomes and Crawford (2004)). This model allows individual behavior to be heterogeneous, with each player following one of a hierarchy of "level-*k*" or "*Lk*"

[6]Rosenthal, Shachat, and Walker (2003) find that QRE *does* explain the qualitative features of subjects' systematic, role-asymmetric deviations from equilibrium in some 2x2 Hide-and-Seek games with varying money payoffs. However, their results were driven by payoff differences, and unrelated to framing. See also McKelvey, Palfrey, and Weber (2000), who find that QRE does less well in explaining behavior in other kinds of zero-sum games.

[7]More precisely: Without differences in magnitudes, the estimated $\lambda \to \infty$. With such differences, for any sufficiently large λ but finite QRE can adjust the perturbations to match the observed frequencies exactly. Thus λ and the perturbations are not identified, but all parameter values that maximize the likelihood are equivalent to those obtained when $\lambda \to \infty$. With finite lambda the estimated perturbations for Hiders (Seekers) are higher (lower) than those estimated for equilibrium with perturbations.

decision rules or *types*; but it assumes that each player's type is drawn from the same distribution, thus eliminating unexplained differences in behavioral assumptions across player roles.[8]

Our level-*k* model has five types, *L0*, *L1*, *L2*, *L3*, and *L4*. Type *Lk* for $k > 0$ anchors its beliefs in a naïve *L0* type, described below, and then adjusts them via thought-experiments involving iterated best responses: *L1* normally best responds to *L0*, *L2* to *L1*, and so on.[9] Each type *Lk* for $k > 0$ also makes uniform errors, with a probability independent of *k* and player role.

Lk types have accurate models of the game and are rational; they depart from equilibrium only in basing their beliefs on simplified models of other players. This yields a workable model of others' choices while avoiding the cognitive complexity of equilibrium analysis. In Selten's (1998) words: "Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties…. Boundedly…rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found. Each step of the procedure is simple, even if many case distinctions by simple criteria may have to be made."

Because *Lk* for $k > 0$ ignores the framing except as it affects the anchoring type *L0*, *L0* is the key to the model's potential to explain RTH's results. *L0* plays multiple roles, as *L1*'s model of others, *L2*'s model of others' models of itself, and so on. We take *L0* to be nonstrategic, as is usual in such analyses; but we allow *L0* Hiders and Seekers to favor the salient focally labeled and/or end locations, to an equal extent in each player role. We also represent *L0*'s preferences directly as choice probabilities rather than payoff perturbations.[10] Our assumption that *L0*'s probabilities are non-uniform departs from much of the literature, but there is ample precedent for adapting *L0* to the setting, and in RTH's games a uniform *L0* would make *Lk* the same as equilibrium for all *k*.[11]

Under these assumptions the level-*k* model can explain RTH's results for plausible values of *L0*'s choice probabilities and the population type frequencies, with no asymmetry across player

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[8]Symmetry in behavioral assumptions across roles is natural here because the types are general decision rules, meant to *explain* differences in Hiders' and Seekers' behavior, and the roles were filled by subjects from the same pool.
[9]Costa-Gomes and Crawford (2004) provide experimental support for our assumptions that *L2* best responds to an *L1* without decision errors, and to *L1* alone rather than a mixture of *L1* and *L0*, etc., unlike in Stahl and Wilson (1995) or Camerer, Ho, and Chong (2004). Our model with five types is the most general model of this kind, because in the unperturbed Hide-and-Seek game our types' choices cycle, so that *L5* is equivalent to *L1*, *L6* to *L2*, and so on (Table II).
[10]This difference from our equilibrium model is less important than it may seem because it does not affect the range of possible best responses for *Lk* for $k > 0$; and *L0*'s choices could be "purified" via privately observed perturbations.
[11]See for example Ho, Camerer, and Weigelt's (1998) analysis of guessing games; or Crawford's (2003) analysis of strategic deception via cheap talk, where the Sender's and Receiver's *L0* types are based on truthfulness or credulity, as in the informal literature on deception. By contrast, the level-*k* model's other basic component, the adjustment of *Lk* beliefs for $k > 0$ via iterated best responses, appears to yield a satisfactory account of initial responses in many settings.

roles in behavioral assumptions. Given *L0*'s attraction to focally labeled and/or end locations, *L1* Hiders choose central A to avoid *L0* Seekers and *L1* Seekers avoid central A in their searches for *L0* Hiders. For similar but more complex reasons, *L2* Hiders choose central A with probability between zero and one and *L2* Seekers choose it with probability one; *L3* Hiders avoid central A and *L3* Seekers choose it with probability between zero and one; and *L4* Hiders and Seekers both avoid central A (Table II). For our estimated type frequencies, which are similar to—but imply somewhat more sophistication than—those that have been estimated for other settings (see Costa-Gomes and Crawford (2004) and the papers discussed there), these choice patterns allow the level-*k* model to explain the prevalence of central A for both Hiders and Seekers, its greater prevalence for Seekers than Hiders, and the fact that Seekers find a Treasure more often than in equilibrium.[12]

To put this explanation into perspective, compare it with a simpler alternative that has been suggested to us: "Hiders feel safer avoiding focal locations, so they are most likely to choose central A; and Seekers know this, so they are also most likely to choose central A." This sounds plausible, but it has two weaknesses: It implicitly assumes that Hiders are systematically less sophisticated (*L1*, in our terminology) than Seekers (*L2*), and it does not explain the greater prevalence of central A for Seekers.[13] Our level-*k* model remedies both weaknesses by using a behaviorally plausible specification of *L0* and the type frequencies, the same for Hiders and Seekers, to explain all three of the robust patterns RTH observed, including the role asymmetries.

Section 5's analysis shows that the level-*k* model is a promising alternative to equilibrium or QRE with payoff perturbations explanations of RTH's results, but all three models' predictions depend on behavioral parameters that must be estimated or translated from other settings. Section 6 compares the models more systematically, focusing on the equilibrium and level-*k* models because when estimated the QRE model effectively reduces to the equilibrium model. We first estimate the equilibrium and level-*k* models econometrically, pooling the data from the six treatments on which

[12]Note that the level-*k* model's explanation goes against its specification of *L0*, which avoids central A in either role. Our level-*k* analysis is similar in some respects to Bacharach and Stahl's (2000) "level-*k* variable-frame" analysis of coordination and Bacharach and Stahl's (1997a) analysis of Hide-and-Seek games (not in the published version, Bacharach and Stahl (2000); and Bacharach and Stahl (1997b), whose title suggests a more detailed version of their Hide-and-Seek analysis, is unavailable). However, our analysis makes much simpler behavioral assumptions, in that only *L0* responds directly to the framing, and our players need not reason about each other's knowledge of *L0*.

[13]Note that levels of sophistication are meaningfully comparable across roles. The quotation in the text could be inverted to "Seekers are drawn to focal locations, so they are unlikely to choose central A; Hiders know this, so they are likely to choose central A." This version uses the same logic as the one in the text, with player roles interchanged; but it does not fit the patterns in RTH's data, suggesting that an explanation requires empirical knowledge in addition to logic.

our analysis focuses. We then address concerns about overfitting by using estimates computed separately, treatment by treatment, to "predict" the results of the other five treatments.

In the equilibrium with payoff perturbations model, the restriction of equal magnitudes across player roles is strongly rejected. With unrestricted perturbations, the equilibrium model has a small likelihood advantage over the level-*k* model, because role-dependent perturbations give it more freedom to fit the observed frequencies. But our favored specification of the level-*k* model has a modest advantage over the equilibrium model in prediction, with 18% lower mean squared error and better predictions in 20 of 30 comparisons.

Overall, our results make a good case for using the level-*k* model to explain RTH's results. It explains the role asymmetries in RTH's data without assuming differences in behavior across roles, while the equilibrium and QRE can only explain them via large, unexplained differences in the signs and magnitudes of Hiders' and Seekers' payoff perturbations. It dispenses with the strong coordination of beliefs assumption that underlies equilibrium and QRE, which is strained for initial responses to a game with subtle differences in payoffs. Finally, it rests on general decision rules and behavioral parameters that, except for *L0*'s choice probabilities, characterize strategic behavior in a way that is portable and stable across games. All these factors give reason for confidence that the level-*k* model explains RTH's results, rather than just describing them.[14]

Section 7 is the conclusion.

2. Cross-Treatment Analogies and Preliminary Tests

This section describes RTH's other treatments in Table I and explains the senses in which they are analogous to the RTH-4 treatment discussed in the Introduction. We argue that the patterns of deviations from equilibrium observed in RTH-4—the prevalence of central A for both Hiders and Seekers, its greater prevalence for Seekers, and the fact that Seekers find the prize more often than in equilibrium—extend, properly interpreted, to the five RTH treatments that are most closely analogous to RTH-4 (Table I, top half). We also report preliminary statistical tests of whether the choice frequencies in those six treatments can be pooled in the econometric analysis.

In RTH-4 the Hider hid a desirable "prize," which we call a "treasure" as in RTH (1996). The five other treatments in the top half of Table I include three more "Treasure" treatments, RT-AABA-Treasure (with the B in the third position, perhaps because it was run in Hebrew), RT-1234-

Treasure, and R-ABAA. They also include two "Mine" treatments, RT-AABA-Mine and RT-1234-Mine, identical to the corresponding Treasure treatments except that the hidden object is undesirable, so that Hiders' and Seekers' payoffs are interchanged. This yields an equivalent normal form with players' roles reversed, leaving equilibrium predictions otherwise unchanged. However, because Hiders inherently move first, even though Seekers do not observe their choices Mine treatments have different extensive forms than Treasure treatments with roles reversed. RTH, suspecting that this difference might make it easier for Seekers to mentally simulate Hiders' choices, used Mine treatments to test whether it explains the role-asymmetric patterns in their data; but the Mine treatments yielded results very close to the corresponding Treasure treatments with roles reversed.[15] This suggests that the role-asymmetries were somehow driven by subjects' responses to the normal-form structure, as in all of the theoretical explanations considered here.

In the three ABAA or AABA Treasure treatments and the AABA Mine treatment, central A—RT's "least salient" location—was the modal choice for both Hiders and Seekers. This pattern extends to the 1234 Treasure and Mine treatments if we follow RT's suggestion that "the least salient response…may correspond to 3, or perhaps 2" and take 2 as analogous to B and 3 to central A. Given this identification, central A was more prevalent for Seekers in all four Treasure treatments and more prevalent for Hiders in both Mine treatments; thus this pattern is also the same in all six treatments if Hiders in Treasure and Seekers in Mine treatments are identified. Further, the frequencies with which Seekers found a Treasure *or* a Mine exceeded ¼, so that Seekers (Hiders) had higher (lower) expected payoffs than in equilibrium in Treasure treatments, and vice versa in Mine treatments. Accordingly, our analysis "builds in" these analogies by identifying 2 with B, and Mine treatments with the corresponding Treasure treatments with reversed player roles. To avoid unnecessary repetition, we use "central A" ("B") to refer to either a central A (B) or a 3 (2) location; and we refer to Mine treatments as if they were Treasure treatments with reversed roles.

After transforming the data as suggested by these identifications, chi-square tests for aggregate differences in subjects' choice frequencies across the six treatments in the top half of Table I reveal no significant difference for Seekers (p-value 0.2297) and a marginally significant difference for Hiders (p-value 0.0417). Pairwise tests show that this last difference can be attributed to RTH-4, which differed from the other treatments in having unusually high frequencies of B (as

[14]Although equilibrium is also a general decision rule, it is less clear how to translate the payoff perturbations needed to explain RTH's results to new games, and we are unaware of any estimates of them to which ours can be compared.
[15]Weber, Camerer, and Knez (2004) found weak effects of timing without observability in other games.

well as central A) for both Hiders and Seekers. Nonetheless, in keeping with our goal of explaining the prevalence of central A for both Hiders and Seekers, and its greater prevalence for Seekers, both of which hold in RTH-4 as well as the other five treatments, we pool all six treatments for the econometric analysis and remark how the results differ for RTH-4. The pooled sample includes 624 Hiders and 560 Seekers, with aggregate choice frequencies as shown in Table III.[16]

Although our analysis focuses on the six treatments in the top half of Table I, the five treatments in the bottom half are worth some discussion because they provide additional evidence of the robustness of the patterns RTH observed. These treatments are all Treasure treatments with the same payoff structure as RTH-4, but with labels with positive or negative connotations and/or focally labeled end locations.[17] RTH-2 and RTH-5 are analogous to RTH-4 except for the connotations of the focal label. RTH-1 and RTH-3 are analogous to RTH-4 except that the focal label is at an end position, and in RTH-3 it has a negative connotation. RTH-6 is analogous to RTH-5 except that the focal label is in the third rather than second position; and is analogous to RTH-2 and RTH-4 except for this difference in position and that the focal label has a positive connotation in RTH-6 but negative or neutral connotations in RTH-2 or RTH-4. The choice frequencies for these imperfectly analogous RTH treatments echo those for the ones we analyze, with complicating shifts in expected directions. It seems likely that Section 5's level-*k* explanation of RTH's results could be adapted to these treatments by estimating new *L0* choice probabilities.

3. Equilibrium with Payoff Perturbations

This section considers explanations of RTH's results via equilibrium analysis of a perturbed game, whose players' payoffs are influenced by the salience of focally labeled and/or end locations as well as their strategic consequences. We take these payoff perturbations to reflect hard-wired preferences, independent of strategic reasoning. Following RTH's discussions of typical responses to salience, we assume that Seekers receive an additional, psychic payoff of e for choosing an end location (equal for both ends for simplicity) or f for choosing the focally labeled B location, and that Hiders lose payoffs of equal magnitudes for such choices (Figure 2). We start by taking e and f to have the same magnitudes for both player roles, and focus on the leading case where $e, f > 0$.[18]

[16]Minor corrections to the published data were needed to reconcile the reported frequencies and sample sizes.
[17]These subjects played a series of games with the same normal form but different labelings. They were anonymously and randomly paired, without feedback, to induce them to treat each of the games as "one-shot."
[18]In Section 5 we report estimates of $e = 0.2187$ and $f = 0.2010$ without this restriction, confirming that $e, f > 0$ provides the best explanation of RTH's results for the equilibrium with restricted perturbations model.

If $-1 < f - 2e$, $2e - 3f$, $2e + f < 3$, the perturbed game has a unique, symmetric, totally mixed equilibrium. In this equilibrium Hiders and Seekers both play A, B, A, A with probabilities $\frac{1}{4} - e/2 + f/4$, $\frac{1}{4} + e/2 - 3f/4$, $\frac{1}{4} + e/2 + f/4$, and $\frac{1}{4} - e/2 + f/4$ respectively. A Hider's expected payoff is $\frac{3}{4} - e/2 - f/4$ and a Seeker's is $\frac{1}{4} + e/2 + f/4$. Thus, both Hiders and Seekers play central A with probability $\frac{1}{4} + e/2 + f/4$, which is greater than $\frac{1}{4}$ whenever $2e + f > 0$ but equal for Hiders and Seekers if e and f are. A Seeker finds the treasure with probability:

(1)

$$m = \left(\frac{1}{4} - \frac{e}{2} + \frac{f}{4}\right)^2 + \left(\frac{1}{4} + \frac{e}{2} - \frac{3f}{4}\right)^2 + \left(\frac{1}{4} + \frac{e}{2} + \frac{f}{4}\right)^2 + \left(\frac{1}{4} - \frac{e}{2} + \frac{f}{4}\right)^2 = \frac{1}{4}\left(1 + (2e - \sqrt{3}f)^2 + 4(\sqrt{3} - 1)ef\right),$$

which is greater than $\frac{1}{4}$ for any $e, f > 0$, so equilibrium with payoff perturbations can explain why Seekers find the treasure more often than in a standard equilibrium. Equilibrium with perturbations of equal magnitudes but opposite signs across player roles can thus explain all the patterns RTH observed except the greater prevalence of central A for Seekers. The model can explain this only by assuming e and f have different magnitudes across roles, with $2e + f$ sufficiently larger for Hiders than Seekers.[19] Such differences give the model enough flexibility to explain almost any pattern.

Hider/Seeker	**A**	**B**	**A**	**A**
A	$0 - e, 1 + e$	$1 - e, 0 + f$	$1 - e, 0$	$1 - e, 0 + e$
B	$1 - f, 0 + e$	$0 - f, 1 + f$	$1 - f, 0$	$1 - f, 0 + e$
A	$1, 0 + e$	$1, 0 + f$	$0, 1$	$1, 0 + e$
A	$1 - e, 0 + e$	$1 - e, 0 + f$	$1 - e, 0$	$0 - e, 1 + e$

Figure 2. Hide-and-Seek Game with Payoff Perturbations

4. Quantal Response Equilibrium with Payoff Perturbations

Because QRE, defined informally in the Introduction, responds only to the payoff structure, it ignores the framing of the Hide-and-Seek game without payoff perturbations (Figure 1). In that game, for any error distribution, there is a unique QRE, which yields the same choice probabilities as equilibrium. To see this, fix a distribution, and suppose to the contrary that in a QRE the most probable location for Hiders, call it L, has probability greater than $\frac{1}{4}$. Because QRE choice probabilities increase with expected payoffs and the game is constant-sum, L must then have the

highest expected payoff for Seekers, and so has probability greater than ¼ for them. But then some location other than L has higher expected payoff for Hiders, contradicting the initial hypothesis.

This section accordingly considers explanations that combine logit QRE with payoff perturbations as in Figure 2, which make QRE sensitive to framing and give it the potential to explain RTH's results. As is almost always the case, such models must be solved computationally.

Figure 3 illustrates logit QRE with payoff perturbations restricted to be equal in magnitude but opposite in sign across player roles, as a function of λ, with $e = 0.2187$ and $f = 0.2010$, the values that best fit RTH's data for Section 2's equilibrium with restricted perturbations model (Section 6). The maximum likelihood estimate of λ in the QRE with restricted perturbations model is infinite, reducing the model to the equilibrium with restricted perturbations model (footnote 7). As in Figure 3, for all combinations of $e, f = 0.1$, 0.2, 0.3, or 0.4 (all consistent with a totally mixed equilibrium), the logit QRE probability of central A dips below 0.25 for low values of λ for Seekers but never for Hiders; and it is always higher for Hiders, reversing the patterns in RTH's data.[20]

Like equilibrium with payoff perturbations, logit QRE can only explain RTH's results by allowing the magnitudes of the perturbations e and f to differ across roles. This yields an effectively infinite estimate of λ, reducing logit QRE, in terms of its substantive implications, to Section 2's equilibrium with unrestricted perturbations model (footnote 7). Figure 4 illustrates logit QRE with $e_H = 0.2910$, $f_H = 0.2535$, and $e_S = f_S = 0.1539$, the values that give the best fit for the equilibrium with unrestricted perturbations model.

5. A Model with Boundedly Rational "Level-*k*" Types

Returning to the Hide-and-Seek game without payoff perturbations (Figure 1), we now consider a structural non-equilibrium model of initial responses with boundedly rational level-k types as discussed in the Introduction. Each player follows one of five types, *L0*, *L1*, *L2*, *L3*, or *L4*, with given probabilities r, s, t, u, and v respectively, the same for each player role. Type *L0* Hiders and Seekers both choose A, B, A, A with probabilities $p/2$, q, $1-p-q$, $p/2$ respectively, taking the probabilities of the end locations to be equal for simplicity. We focus on the leading case in which $p > ½$ and $q > ¼$, reflecting *L0*'s nonstrategic attraction to salient locations. We also assume that for $k > 0$, an *Lk* Hider or Seeker normally chooses type k's location for his role, defined by iterating

[19] $2e+f$ must be larger for Hiders because larger values are associated with higher probabilities of central A, and Hiders' value determines Seekers' probability. In Section 5 we report estimates of $e_H = 0.2910$, $f_H = 0.2535$, and $e_S = f_S = 0.1539$ for the equilibrium with unrestricted perturbations model: all positive as expected, and nearly twice as large for Hiders.
[20] There is enough structure to suggest that this result is symptomatic of a theorem, but we have been unable to prove it.

best responses to *L0*, with ties broken randomly with equal probabilities.[21] But with probability ε, equal across roles and values of $k > 0$, he makes an error, in which case he chooses each location with probability ¼; these errors are independently and identically distributed across players. This implies that a location can be chosen with probability greater than ¼ in the population of Hiders or Seekers only if it is a best response for some type that has positive probability in the population.

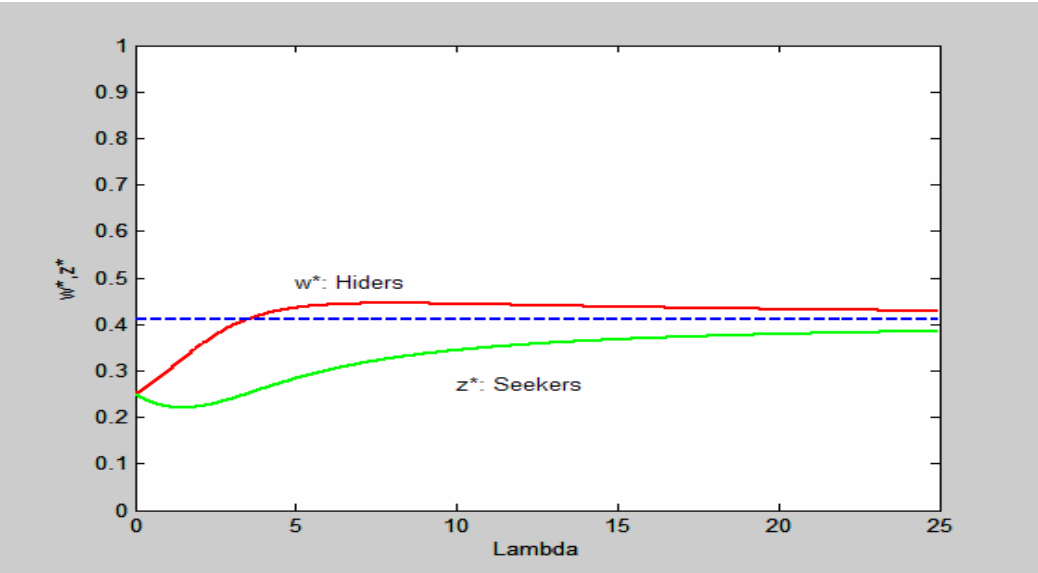

Figure 3. QRE with Payoff Perturbations of Equal Magnitudes Across Player Roles

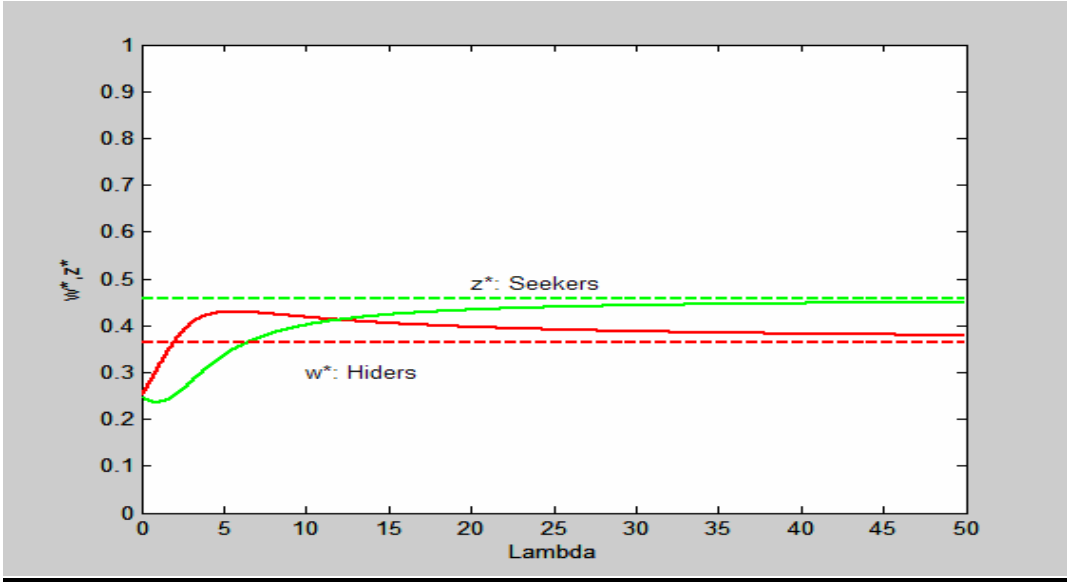

Figure 4. QRE with Payoff Perturbations of Differing Magnitudes Across Player Roles

[21] As noted in the Introduction, *L0* plays multiple roles in this model, as *L1*'s model of others, *L2*'s model of others' model of itself, etc. This is a plausible simplification if the intuitions *L0* reflects are widely understood. It is less strained than the knowledge assumptions of equilibrium with payoff perturbations because it refers only to *L0*, which reflects more basic intuitions; and because here there is no need for the knowledge about *L0* to be mutual or common.

Table II lists *L0*'s, *L1*'s, *L2*'s, *L3*'s, and *L4*'s normal choice probabilities when $p + 2q > 1$ and $3p + 2q > 2$, distinguishing the regions $p < 2q$ and $p > 2q$.[22] These inequalities hold when *L0* favors both B and end locations, so that $p > ½$ and $q > ¼$, and also for some other values of p and q. It is easy to show that *L5*'s choice probabilities (though not its expected payoffs) are the same as *L1*'s, so that the types cycle after *L4* (footnote 9). The bottom lines of Table II give the population's total choice probabilities for the entire population in this case, including errors. Figure 5 summarizes the types' normal choices for the other cases, where it is not assumed that $p > ½$ and $q > ¼$.

For central A to be prevalent for both Hiders and Seekers, its total probability for Hiders,

(2) $$r(1-p-q)+(1-\varepsilon)[s+(t/2 \text{ if } p > 2q) + (t/3 \text{ if } p < 2q)]+(1-r)\varepsilon/4 > ¼.$$

This means that a weighted average of the population frequencies of *L1* and *L2* (with weights depending on whether $p > 2q$, and on ε) is sufficiently greater than the frequency of *L0* to outweigh *L0* Hiders' tendency to avoid central A. For central A to be more prevalent for Seekers than Hiders,

(3) $$t + u > 2s \text{ if } p > 2q \text{ or } 2t + u > 3s \text{ if } p < 2q.$$

This means that a weighted average of the population frequencies of *L2* and *L3* (with weights depending on whether $p > 2q$) is sufficiently greater than the frequency of *L1* to outweigh *L1* Seekers' tendency to avoid central A. Inequalities (2) and (3) show that the level-*k* model can explain the prevalence of central A for both Hiders and Seekers and its greater prevalence for Seekers than Hiders for values of the population type frequencies similar to, but somewhat more sophisticated than, those estimated for other settings (see Costa-Gomes and Crawford (2004) and the papers discussed there). It is noteworthy that RTH's results, when viewed through the lens of the non-equilibrium level-*k* model via (2) and (3), imply *lower* bounds on the population's sophistication, in contrast to RTH's interpretation of the results as evidence of strategic naivete.

6. Econometric Analysis

The models considered here all make predictions that depend on behavioral parameters: payoff perturbations for equilibrium and QRE, a precision parameter for QRE, and *L0*'s choice probabilities and the population type frequencies for the level-*k* model. This section makes our

[22]Given our error structure, there are linear dependencies among types' choice probabilities for Hiders and Seekers; but they are different for Hiders and Seekers, so the population type frequencies are still identified from the full sample.

Table II. Types' Expected Payoffs and Choice Probabilities when $p + 2q > 1$ and $3p + 2q > 2$

Hider	Exp. Payoff	Choice Pr.	Exp. Payoff	Choice Pr.	Seeker	Exp.	Choice Pr.	Exp. Payoff	Choice Pr.
	$p < 2q$	$p < 2q$	$p > 2q$	$p > 2q$		$p < 2q$	$p < 2q$	$p > 2q$	$p > 2q$
***L0* (Pr. *r*)**					***L0* (Pr. *r*)**				
A	-	$p/2$	-	$p/2$	A	-	$p/2$	-	$p/2$
B	-	q	-	q	B	-	q	-	q
A	-	$1-p-q$	-	$1-p-q$	A	-	$1-p-q$	-	$1-p-q$
A	-	$p/2$	-	$p/2$	A	-	$p/2$	-	$p/2$
***L1* (Pr. *s*)**					***L1* (Pr. *s*)**				
A	$1 - p/2 < 3/4$	0	$1 - p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	$1/2$
B	$1 - q < 3/4$	0	$1 - q < 3/4$	0	B	$q > 1/4$	1	$q > 1/4$	0
A	$p + q > 3/4$	1	$p + q > 3/4$	1	A	$1-p-q < 1/4$	0	$1-p-q < 1/4$	0
A	$1 - p/2 < 3/4$	0	$1 - p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	$1/2$
***L2* (Pr. *t*)**					***L2* (Pr. *t*)**				
A	1	1/3	$1/2$	0	A	0	0	0	0
B	0	0	1	$1/2$	B	0	0	0	0
A	1	1/3	1	$1/2$	A	1	1	1	1
A	1	1/3	$1/2$	0	A	0	0	0	0
***L3* (Pr. *u*)**					***L3* (Pr. *u*)**				
A	1	1/3	1	1/3	A	1/3	1/3	0	0
B	1	1/3	1	1/3	B	0	0	$1/2$	$1/2$
A	0	0	0	0	A	1/3	1/3	$1/2$	$1/2$
A	1	1/3	1	1/3	A	1/3	1/3	0	0
***L4* (Pr. *v*)**					***L4* (Pr. *v*)**				
A	2/3	0	1	$1/2$	A	1/3	1/3	1/3	1/3
B	1	1	$1/2$	0	B	1/3	1/3	1/3	1/3
A	2/3	0	$1/2$	0	A	0	0	0	0
A	2/3	0	1	$1/2$	A	1/3	1/3	1/3	1/3

Total Probability		$p < 2q$	$p > 2q$	**Total Probability**		$p < 2q$	$p > 2q$
A	$rp/2+(1-\varepsilon)[t/3+u/3]+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[u/3+v/2]+(1-r)\varepsilon/4$		A	$rp/2+(1-\varepsilon)[u/3+v/3]+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[s/2+v/3]+(1-r)\varepsilon/4$	
B	$rq+(1-\varepsilon)[u/3+v]+(1-r)\varepsilon/4$	$rq+(1-\varepsilon)[t/2+u/3]+(1-r)\varepsilon/4$		B	$rq+(1-\varepsilon)[s+v/3]+(1-r)\varepsilon/4$	$rq+(1-\varepsilon)[u/2+v/3]+(1-r)\varepsilon/4$	
A	$r(1-p-q)+(1-\varepsilon)[s+t/3]+(1-r)\varepsilon/4$	$r(1-p-q)+(1-\varepsilon)[s+t/2]+(1-r)\varepsilon/4$		A	$r(1-p-q)+(1-\varepsilon)[t+u/3]+(1-r)\varepsilon/4$	$r(1-p-q)+(1-\varepsilon)[t+u/2]+(1-r)\varepsilon/4$	
A	$rp/2+(1-\varepsilon)[t/3+u/3]+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[u/3+v/2]+(1-r)\varepsilon/4$		A	$rp/2+(1-\varepsilon)[u/3+v/3]+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[s/2+v/3]+(1-r)\varepsilon/4$	

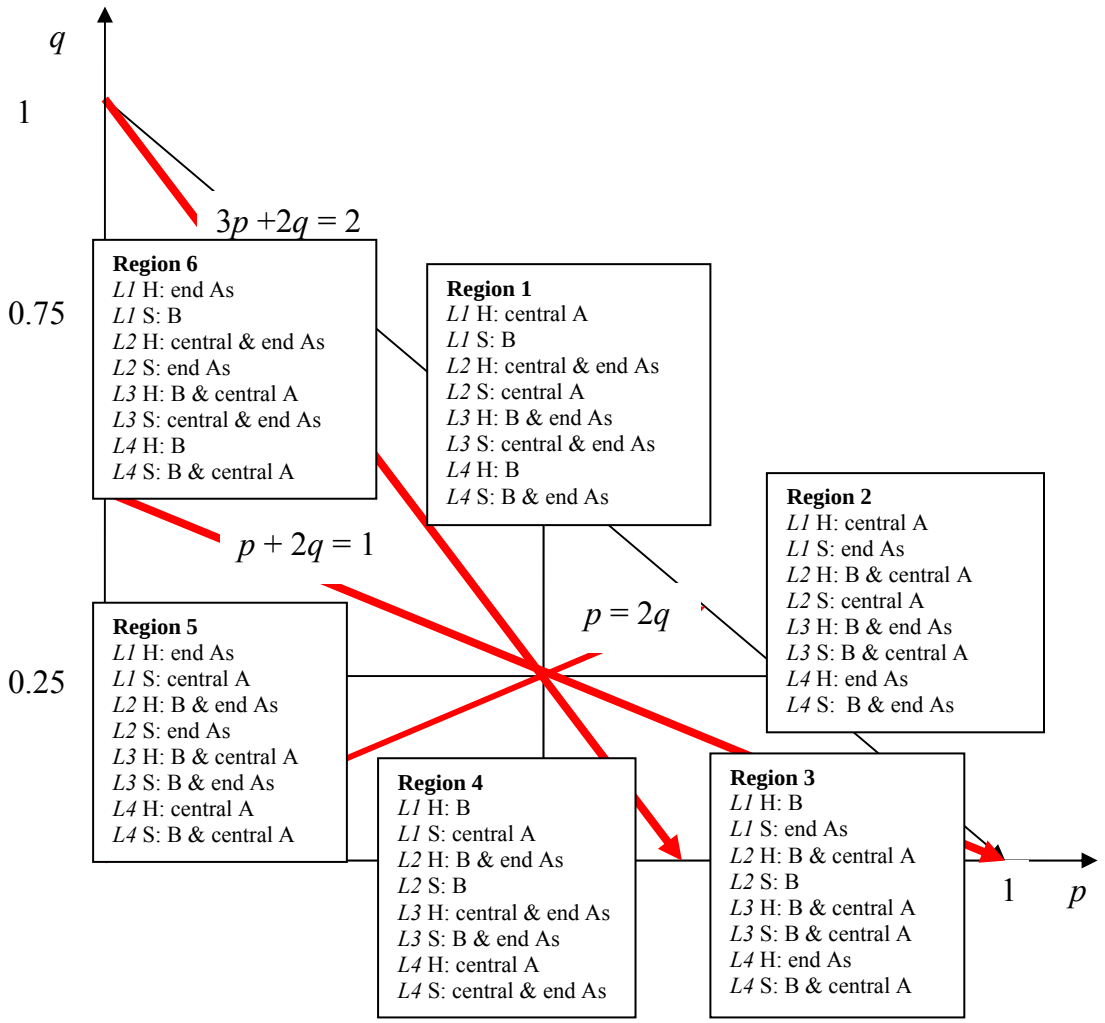

Figure 5. *L1*'s Through *L4*'s Choices as Functions of *L0*'s Choice Probabilities

analyses more concrete by using RTH's data to estimate those parameters econometrically. We focus on the equilibrium and level-*k* models because the estimated QRE model reduces to equilibrium. We then address the issue of overfitting by using estimates computed for each model, treatment by treatment, to "predict" the results of the other five treatments.

Our purpose is to illustrate the possibilities of the level-*k* and equilibrium with perturbations models, not to take a definitive position on the behavioral parameters, which in our view would require more comprehensive experiments, perhaps with a design that tracks individual subjects' choices across different but related games as in Stahl and Wilson (1994, 1995); Costa-Gomes, Crawford, and Broseta (2001); or Costa-Gomes and Crawford (2004). Because both models have enough flexibility to fit the observed choice frequencies very well, our econometrics amount to

little more than a calibration exercise. But using econometrics to calibrate the models constrains our discretion, and yields likelihoods that provide an objective criterion by which to compare them.

Our econometric model is a mixture-of-types model as in Stahl and Wilson (1994, 1995) or Costa-Gomes, Crawford, and Broseta (2001), with one type for the equilibrium with perturbations model and five for the level-k model (Sections 2 and 4). Let X_{ij} denote the total numbers of Hiders or Seekers (as $i = H, S$) who choose location j (with A, B, A, A denoted 1, 2, 3, 4). If p_{ijk} is the probability that a Hider or Seeker of type k chooses location j, the likelihood can be written:

$$L \equiv \prod_{i=H,S} \prod_{j=1,2,3,4} \prod_{k} [p_{ijk}]^{X_{ij}}. \tag{4}$$

The p_{ijk} for the equilibrium model are $\frac{1}{4} - e/2 + f/4$, $\frac{1}{4} + e/2 - 3f/4$, $\frac{1}{4} + e/2 + f/4$, and $\frac{1}{4} - e/2 + f/4$ for locations A, B, A, A ≡ 1, 2, 3, 4, for $i = H, S$ (Section 3). Because the equilibrium that best describes RTH's results will be totally mixed, here we dispense with an error structure.

The p_{ijk} for the level-k model can be read from Figure 5 (where multiple listed locations for Hiders or Seekers in a region have equal choice probabilities) or, when $p > \frac{1}{2}$ and $q > \frac{1}{4}$, Table 2. Here we specify the error structure as simply as possible (uniform) to avoid specification bias.

Table III summarizes parameter estimates and likelihoods for the equilibrium with payoff perturbations and level-k models, the former with and without the restriction that the perturbations are equal in magnitude for Hiders and Seekers, and the latter estimated under the constraints $p > \frac{1}{2}$ and $q > \frac{1}{4}$, which ensure that $L0$ favors focally labeled or end locations.[23] The table also gives the models' predicted choice frequencies, with RTH's observed frequencies for comparison.

For the equilibrium model, we strongly reject the restrictions that $e_H = e_S$ and $f_H = f_S$ (p-value 0.0022). Accordingly, we focus on the equilibrium model with unrestricted perturbations. Both it and the level-k model have enough flexibility to fit the observed choice frequencies very well, except that both restrict the two end locations to have equal probabilities, which is not quite true in the data. The unrestricted equilibrium model has a small likelihood advantage, because role-dependent perturbations give it slightly more freedom to fit RTH's observed choice frequencies.[24]

[23]These constraints are binding. Estimating the model without them yields estimates in Region 5 (Figure 5), with $p < \frac{1}{2}$, $q < \frac{1}{4}$, $2q > p$, and a fit nearly identical to that of equilibrium with unrestricted perturbations. We focus on the estimates in the text because $p < \frac{1}{2}$, $q < \frac{1}{4}$ is implausible for a nonstrategic $L0$, and such estimates allow the level-k model to overfit, creating the same "prediction" problems we identify below for equilibrium with unrestricted perturbations.
[24]To put this difference in perspective, the maximum possible log-likelihood, associated with perfect prediction of the choice frequencies, is –1561.7 (strictly negative here because the model makes probabilistic predictions).

We are unaware of any estimates with which to compare its estimated payoff perturbations, but their signs are behaviorally plausible; however, their magnitudes are nearly twice as large for Hiders than Seekers, a difference for which it seems hard to find a sensible explanation.

Table III. Parameter Estimates and Likelihoods for the Leading Models

Model	Ln L	Parameter Estimates		Actual and Predicted Choice Frequencies for Hiders and Seekers			
				A	**B**	**A**	**A**
Observed choice frequencies			**H**	0.2163	0.2115	0.3654	0.2067
			S	0.1821	0.2054	0.4589	0.1536
Equilibrium without perturbations/random	-1641.4		**H**	0.2500	0.2500	0.2500	0.2500
			S	0.2500	0.2500	0.2500	0.2500
Equilibrium with perturbations of equal magnitudes across player roles	-1568.5	$e_H = e_S = 0.2187$	**H**	0.1897	0.2085	0.4122	0.1897
		$f_H = f_S = 0.2010$	**S**	0.1897	0.2085	0.4122	0.1897
Equilibrium with perturbations of unrestricted magnitudes across player roles	-1562.4	$e_H = 0.2910, f_H = 0.2535$	**H**	0.2115	0.2115	0.3654	0.2115
		$e_S = 0.1539, f_S = 0.1539$	**S**	0.1679	0.2054	0.4590	0.1679
Level-*k* with $p > ½$, $q > ¼$, and $p > 2q$	-1564.4	$r = 0$, $s = 0.1896$, $t = 0.3185$, $u = 0.2446$, $v = 0.2473$, $\varepsilon = 0$	**H**	0.2052	0.2408	0.3488	0.2052
			S	0.1772	0.2047	0.4408	0.1772

The level-*k* model's estimates are generally behaviorally plausible, with $r = 0$, so that *L0* exists only in the minds of *L1* through *L4*; and $\varepsilon = 0$, so there are no choices unexplained by *L1* through *L4*.[25] Because the estimated $r = 0$, *L0*'s choice probabilities p and q are not identified; but the region in which the likelihood is maximized subject to $p > ½$ and $q > ¼$ is identified as Region 2 (Figure 5), where $p > 2q$ so *L0* favors end more than focally labeled locations.[26] The estimated

[25]The finding that there are no *L0* subjects is consistent with the common finding that people systematically underestimate others' sophistication relative to their own (see for instance Weizsäcker (2003)). In Hide-and-Seek without payoff perturbations, our uniform errors are perfectly confounded with the equilibrium mixed-strategy probabilities. Thus our finding that $\varepsilon = 0$ also suggests the absence of an *Equilibrium* type. Further, we can reject explanations in which a not-too-large part of the population choose locations with given probabilities (like *L0*) and the rest play equilibrium in a game among themselves, taking the first part's behavior into account, like the *Sophisticated* type in Costa-Gomes, Crawford, and Broseta (2001), Crawford (2003), or Costa-Gomes and Crawford (2004).
[26]The maximized log-likelihood in Region 1 is slightly lower, at -1567.9. In a sense the "best fit" is on the border between Regions 1 and 2, where $p = 2q$, which yields log-likelihood -1563.8; but here the parameters are not identified

type frequencies are close to previous estimates, but somewhat more sophisticated than usual (Stahl and Wilson (1994); Costa-Gomes, Crawford, and Broseta (2001); Costa-Gomes and Crawford (2004); Camerer, Ho, and Chong (2004)).[27] This could be due either to more sophisticated subject pools, or to the simplicity and transparency of the strategic questions Hide-and-Seek poses.

Given the flexibility of the equilibrium with unrestricted perturbations and level-*k* models' parameterizations, overfitting is a concern. We test for it by re-estimating the models separately for each of the six treatments and using each re-estimated model to "predict" the observed choice frequencies of the other five treatments. For the level-*k* model we restrict the estimates to Region 2 ($p > 2q$) because the parameters are not identified in Region 1 ($p < 2q$) without imposing $r = \varepsilon = 0$; and while the fit is slightly better in Region 1 for RTH-4 (log-likelihood -143.0 versus -144.2 in Region 2), RT-AABA-Treasure (-361.1 versus -363.6), and R-ABAA (-142.0 versus -142.1), Region 2 yields more sensible parameter estimates and better predictions. We evaluate goodness-of-fit by the mean squared deviation ("MSD") between predicted and observed choice frequencies.

Tables IV-VII summarize the results of the overfitting test. Although the equilibrium with unrestricted perturbations model has a fit at least as good in each treatment (Table VII's diagonal), our favored specification of the level-*k* model has a modest advantage in prediction, with mean squared prediction error 18% lower and better predictions in 20 of 30 comparisons (Table VII).[28]

7. Conclusion

This paper has compared alternative explanations of the systematic patterns of deviation from equilibrium observed in RTH's experiments with two-person constant-sum Hide-and-Seek games with unique mixed-strategy equilibria and non-neutral framing of locations. We focus on two models, equilibrium with unrestricted payoff perturbations and a structural non-equilibrium model of initial responses based on boundedly rational "level-*k*" types. A third model, logit QRE with payoff perturbations, reduces when estimated to the equilibrium with perturbations model.

(in RTH's dataset) even if we restrict $r = \varepsilon = 0$. The parameters are identified if we restrict $r = s = v = 0$, allowing $\varepsilon > 0$, which yields the model with role-dependent *L0* of the June 2004 preliminary draft of this paper. Due to these identification problems and the theoretical advantages of role-independent *L0*, we focus here on the interiors of regions.

[27]When the estimates are computed separately for RTH-4 (the treatment for which we found a marginally significant difference) and the other five treatments pooled, we find that RTH-4's likelihood (subject to $p > ½$ and $q > ¼$) slightly favors Region 1 ($p < 2q$) over Region 2 (log-likelihood -143.0 versus -144.2), with very different type frequencies; while the pooled treatments' likelihood slightly favors Region 2 (log-likelihood -1415.0 versus -1417.8).

[28]However, if we allow Region 1 estimates for RTH-4, RT-AABA-Treasure, and R-ABAA and assume $r=\varepsilon=0$ to avoid identification problems, the level-*k* and equilibrium with unrestricted perturbations models have the same overall MSD.

Table IV. Treatment by Treatment Parameter Estimates

Treatment	Level-*k*						Equilibrium with Perturbations			
	r	s	t	u	v	ε	e_H	f_H	e_S	f_S
RTH-4	0	0.2499	0.2643	0.4858	0.0000	0	0.3307	0.1451	0.2736	0.0377
RT-AABA-Treasure	0	0.1577	0.3265	0.3226	0.1932	0	0.3648	0.2941	0.1164	0.1640
RT-AABA-Mine	0	0.1566	0.3393	0.0686	0.4355	0	0.1818	0.2121	0.1028	0.2192
RT-1234-Treasure	0	0.1572	0.3810	0.1421	0.3197	0	0.3035	0.2976	0.1471	0.1390
RT-1234-Mine	0	0.2066	0.3153	0.2603	0.2178	0	0.2669	0.2406	0.1667	0.1111
R-ABAA	0	0.1933	0.3743	0.2683	0.1641	0	0.4141	0.3594	0.2500	0.2600

Table V. Level-*k* MSDs Treatment by Treatment

Overall MSD 0.00343	**RTH-4**	**RT-AABA-Treasure**	**RT-AABA-Mine**	**RT-1234-Treasure**	**RT-1234-Mine**	**R-ABAA**
RTH-4	0.0020	0.0032	0.0098	0.0031	0.0019	0.0032
RT-AABA-Treasure	0.0047	0.0014	0.0061	0.0016	0.0009	0.0039
RT-AABA-Mine	0.0132	0.0042	0.0011	0.0029	0.0023	0.0085
RT-1234-Treasure	0.0072	0.0016	0.0029	0.0007	0.0002	0.0037
RT-1234-Mine	0.0054	0.0017	0.0035	0.0009	0.0000	0.0034
R-ABAA	0.0040	0.0023	0.0073	0.0016	0.0010	0.0023

Table VI. Equilibrium with Perturbations MSDs Treatment by Treatment

Overall MSD 0.00418	**RTH-4**	**RT-AABA-Treasure**	**RT-AABA-Mine**	**RT-1234-Treasure**	**RT-1234-Mine**	**R-ABAA**
RTH-4	0.0002	0.0088	0.0156	0.0079	0.0050	0.0087
RT-AABA-Treasure	0.0089	0.0001	0.0039	0.0013	0.0017	0.0022
RT-AABA-Mine	0.0153	0.0034	0.0005	0.0031	0.0032	0.0070
RT-1234-Treasure	0.0076	0.0009	0.0031	0.0005	0.0004	0.0025
RT-1234-Mine	0.0053	0.0018	0.0037	0.0009	0.0000	0.0036
R-ABAA	0.0085	0.0019	0.0071	0.0027	0.0032	0.0004

Table VII. Difference in MSDs (Equilibrium – Level-*k*) Treatment by Treatment

Overall MSD Difference 0.00075	**RTH-4**	**RT-AABA-Treasure**	**RT-AABA-Mine**	**RT-1234-Treasure**	**RT-1234-Mine**	**R-ABAA**
RTH-4	-0.0018	0.0056	0.0058	0.0048	0.0031	0.0055
RT-AABA-Treasure	0.0042	-0.0013	-0.0022	-0.0003	0.0009	-0.0017
RT-AABA-Mine	0.0021	-0.0008	-0.0006	0.0001	0.0008	-0.0015
RT-1234-Treasure	0.0003	-0.0007	0.0001	-0.0002	0.0002	-0.0012
RT-1234-Mine	-0.0002	0.0001	0.0001	0.0001	-0.0000	0.0002
R-ABAA	0.0045	-0.0004	-0.0001	0.0010	0.0023	-0.0020

The predictions of the equilibrium with perturbations and level-*k* models both depend on behavioral parameters. When these are estimated econometrically, both models prove flexible enough to fit RTH's observed choice frequencies very well. Equilibrium with payoff perturbations has a small advantage in likelihood, but the level-*k* model has a modest advantage with regard to overfitting, with cross-treatment "predictions" with 18% lower mean squared errors on average.

Overall, our results make a good case for using the level-*k* model to explain RTH's results. It explains the role asymmetries in RTH's data without assuming differences in behavior across roles, while equilibrium with payoff perturbations requires large, unexplained differences in the magnitudes of Hiders' and Seekers' perturbations. It dispenses with the strong coordination of beliefs assumption that underlies the equilibrium with perturbations model, which seems strained for initial responses to a game with subtle differences in payoffs. Finally, the level-*k* model rests on general decision rules and behavioral parameters that, with the exception that *L0* may need to be tailored to each new game, can be used directly to predict behavior in other games, and it has a modest advantage in cross-treatment "predictions" in RTH's dataset. These considerations all suggest that the level-*k* model better identifies the structure of subjects' responses to RTH's Hide-and-Seek games than the alternatives considered here.

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